

TWO ESSAYS ON THE LOW VOLATILITY ANOMALY

By

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## Chapter One: Dissecting the Low Volatility Anomaly

### Introduction

The low volatility anomaly questions a fundamental aspect of financial theory. The anomaly claims that stocks with the lowest volatility have the highest returns and stocks with the highest volatility have the lowest returns. While financial theory suggests a positive relationship between risk and return, the low volatility anomaly indicates the opposite. A direct implication of this anomaly is that investing in low volatility stocks and shorting high volatility stocks will produce large risk-adjusted returns.

Many investment funds designed to track the performance of low volatility stocks have started over the last few years. Three new low volatility ETFs (Russell 1000 Low Volatility, Russell 2000 Low Volatility, and PowerShares S&P 500 Low Volatility) all launched within three weeks of one another in May 2011. The Powershares ETF alone amassed about \$300 million in capital in its first five months of operation.<sup>1</sup> But despite its current popularity within the investment community, the low volatility anomaly is still not well understood.

I analyze the low volatility anomaly from 1980 and 2011 and find it produces large returns. One dollar invested in an equal weighted portfolio of low volatility stocks in July 1980 is worth \$89.50 at the end of 2011. The matching portfolio of high volatility stocks is worth only \$4.84. The difference in performance is even more evident once the riskiness of each portfolio is taken into account. The Sharpe and Treynor ratios of the low volatility stock portfolio are about 5x those of the high volatility stock portfolio.

I note that portfolio choices made on the basis of volatility implicitly sort on other criteria as well. I find that low volatility stocks are typically low beta, high capitalization,

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<sup>1</sup> See *Beat the Market – With Less Risk*, Wall Street Journal, October 1, 2011

value firms. On the other hand, high volatility stocks are typically high beta, low capitalization, growth firms. While these characteristics do not explain the difference in return between high and low volatility stocks, I do find the strength of the low volatility anomaly varies across these measures. The anomaly is present in all but the smallest of U.S. equities and is much stronger among growth stocks than value stocks.

Because prior work on the anomaly has used many different method for measuring volatility, I test portfolios formed on total, idiosyncratic, and systematic volatility, measuring each over numerous time periods. I find that portfolios formed on short-term measures of total and idiosyncratic volatility produce the largest risk-adjusted returns. Subsequently, I regress future returns on past measures of volatility and determine that idiosyncratic volatility, not total or systematic volatility, is the primary driver of the low volatility anomaly. Being one standard deviation above the mean idiosyncratic volatility measured over the past six months decreases return in following month by about 0.30%.

While idiosyncratic volatility is responsible for the anomaly, I find no dominant form. Instead I show that daily idiosyncratic volatility over the past month and monthly idiosyncratic volatility over the past six months together most effectively capture the anomaly. A low volatility portfolio based on both measures of idiosyncratic volatility produces a Fama-French four-factor alpha about 7.8% per year greater than that of the matching high volatility portfolio. I find this difference in alpha is over 1.4% per month during the first half of my sample, but the difference is less than 2.0% per year in the second half.

However, I do not find that the low volatility has disappeared. I instead show that it is hidden by misspecification in the pricing model. A portfolio of low volatility stocks

still outperforms a portfolio of high volatility stocks on a raw return basis from 1996 through 2011, but the effect of the formation and collapse of multiple market bubbles, e.g., the dot-com bubble and 2008 financial crisis, requires model modification. During the formation of these bubbles high volatility stocks perform well and get systematically sorted into the past winner portion of the Fama-French momentum factor (UMD). But when a bubble collapses, high volatility stocks sustain large losses and are quickly sorted into the past loser portion of the factor. This process creates a nonlinear relationship between the momentum factor and the returns on high volatility stocks.

After adjusting the momentum factor for my sample and adding a squared term to the Fama-French four-factor model, I find a large difference in risk-adjusted performance between low and high volatility stocks exists from 1996 through 2011. The difference in alpha between the low and high volatility stocks changes from 0.14% per month with the original specification to 0.98% per month after modification. The linear momentum of the high volatility stock portfolio is negative, but the squared term captures a large positive effect. The portfolio of low volatility stocks has little linear or nonlinear momentum exposure.

## **Literature Review**

### *The relationship between risk and return*

The canonical relationship between risk and return is ubiquitous. Sharpe (1964) and Lintner (1965) demonstrated that only the systematic portion of an asset's risk, derived from an asset's relationship with the market portfolio, should be priced. Any idiosyncratic risks should not be priced because they can be inexpensively diversified

away. Fama and Macbeth (1973) verified this theory empirically by showing stock returns are linearly increasing in their exposure to systematic risk, i.e., beta, and that idiosyncratic risk was not priced.

However, it is now well known that stock returns cannot be explained by beta alone. Fama and French (1992, 1993) show that size and market-to-book ratio explain the cross-section of stock returns better than beta. Jegadeesh and Titman (1993) find stocks have momentum, i.e., stocks that performed well in the recent past will continue to perform well in future. Fama and French (2008) show that net stock issuance and accruals are also predictive of future returns.

It is clear these variable help explain the cross-section of returns, but it is less clear if they proxy for systematic risks. Jegadeesh and Titman (2001) claim that momentum is generated by delayed overreactions in the market. Lakonishok, Shleifer, and Vishny (1994) find that high book-to-market firms outperform low book-to-market firms regardless of market conditions. On the other hand, Fama (1998) claims that anomalies are chance results, overreactions are as common as underreactions, which are sensitive to method of measurement.

Further, relaxing the assumptions underlying systematic risk theory shows that idiosyncratic risk should itself be rationally priced. Merton (1987) claims that idiosyncratic risk should be priced if investors' portfolios are under-diversified, in this case due to information gathering costs. In a similar model, Hirshleifer (1988) shows that idiosyncratic risk should be priced if investors face fixed costs to participate in a market. In both instances, systematic risk will still matter, but returns should also increase with exposure to idiosyncratic risk. Alternatively, Miller (1977) argues that if investors have

heterogeneous estimates of future firm performance and are restricted in their ability to short firms, then there should be a negative relationship between return and idiosyncratic risk.

### *The relationship between volatility and return*

Early empirical tests of the volatility anomaly focused on total, not idiosyncratic, risk. Haugen and Heins (1972) and Haugen and Heins (1975) first showed that firms with a low standard deviation of past returns outperformed those with a high standard deviation. Baker and Haugen (2012) and Blitz and Vliet (2007) demonstrate that this effect is present in equity markets throughout the world. While these results are surprising, it is unclear what the result is capturing. Total risk is positively correlated with idiosyncratic risk among stocks. Most papers instead focus directly on idiosyncratic risk.

The most common method of determining a firm's idiosyncratic volatility is described in Ang, Hodrick, Xing, and Xiang (2006, 2009). They calculate idiosyncratic volatility for a stock as the standard deviation of the residuals from regressing a firm's daily returns over the past month on the three Fama and French (1993) factors and a momentum factor. Consistent with Miller (1977), firms with high idiosyncratic volatility have lower returns. This measure is questioned by Bali and Cakici (2008). They claim that the relationship is not robust to changes in data frequency, weighting scheme, and sample. On the other hand, Clarke, De Silva, and Thorley (2010) form a pricing factor VMS, volatile minus stable, based on idiosyncratic volatility and find it explains a large portion of the covariance of stock returns.

A third method used to measure a firm's idiosyncratic volatility is proposed by Fu (2009). He finds that past idiosyncratic volatility is a poor proxy for future idiosyncratic volatility. Instead, he uses an exponential GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model to estimate a stock's expected idiosyncratic volatility. Consistent with Merton (1987) and Hirshleifer (1988), he finds returns increase with expected idiosyncratic volatility. Huang, Liu, Rhee, and Zhang (2010) perform a similar analysis and come to the same conclusion.

#### *Explanations for the volatility anomaly*

One potential explanation for the anomaly is investor preference for lottery-type payoffs. Shefrin and Statman (2000) create a model where investors mentally divide their portfolio into two groups, bonds and lottery tickets. They will overprice and accept low expected returns from highly idiosyncratic assets if the possibility of very large payoffs exists. Barberis and Huang (2008) claim that this investor desire for asymmetric payoffs causes return skewness to be priced by the market. Boyer, Mitton, and Vornick (2010) confirm that returns are negatively associated with a stock's expected idiosyncratic skewness. Bali, Cakici, and Whitelaw (2011) proxy for lottery payouts using the maximum daily return for a firm in the prior month and find controlling for lottery preference creates a positive relationship between idiosyncratic volatility and return. Han and Kumar (2012) measure the market's view of a stock as a lottery ticket by measuring the amount trading in the stock done by retail investors. They find stocks with trading dominated by retail investors have low abnormal returns.

Fu (2009) and Huang, Liu, Rhee, and Zhang (2010) both claim that the negative relationship between idiosyncratic volatility and return is related to short-term return reversals. They control for a stock's return during the past month in their models and find idiosyncratic volatility then has little effect. In addition, these reversals are concentrated in smaller stocks, calling into question the economic relevance of the anomaly. As mentioned before, they both find a positive relationship between idiosyncratic volatility and return with their expected idiosyncratic volatility specification. On the other hand, Chen, Jiang, Xu, and Yao (2012) find return reversals do not explain the negative relationship between idiosyncratic volatility and return. They also show that the anomaly is robust to excluding microcaps.

Other explanations for the volatility anomaly include Wong (2011), who finds that accounting for a combination of earnings shocks and earnings momentum eliminates the idiosyncratic volatility anomaly; Hsu, Kudoh, and Yamada (2012), who show that inflated earnings forecasts from sell-side analysts cause high volatility stocks to become overpriced and have low returns; Johnson (2004), who claims that unpriced information risk (proxied by analyst forecast dispersion) causes a negative relationship between return and idiosyncratic risk even without market frictions or irrational agents; Avramov, Chordia, Jostova, and Philipov (2012), who find that investment strategies taking advantage of the anomaly derive their profits solely from positions in financially distressed firms; and Chen and Petkova (2012), who show that only one component of idiosyncratic volatility, the average variance risk, is priced.

The literature is still unsettled as to the root cause the volatility anomaly. Hou and Loh (2012) simultaneously test many of the prior explanations and find that lottery



preferences, short-term return reversals, and earnings shocks together explain 60 to 80% of the negative relationship between returns and idiosyncratic volatility. Lottery preference alone can explain 48 to 67% of the relationship.

Limits to arbitrage are often cited as impediments to an efficient market correcting the anomaly. Baker, Bradley, and Wurgler (2011) claim that large institutions cannot take advantage of low volatility stocks because of leverage limits and requirements to beat benchmarks. Frazzini and Pedersen (2013) show an investor in a similar environment will also overprice high beta stocks. Boehme, Danielson, Kumar, and Sorescu (2009) show that short selling constraints will also cause a negative relationship between return and idiosyncratic volatility. Garcia-Feijoo, Li, and Sullivan (2012) demonstrate that low volatility strategies require a large amount of trading in stocks with low liquidity. Han and Lesmond (2011) find that after accounting for multiple microstructure effects the relationship between return and idiosyncratic volatility is gone.

## **Data and Methods**

I use the CRSP stock files to build my sample of firms. I use only ordinary shares (CRSP share codes 10 and 11) that trade on the NYSE, NASDAQ, or AMEX (CRSP exchange codes 1, 2, and 3). I consider a stock a penny stock and omit it from the sample until its price exceeds \$5 at the end of a month. From that point forward, it remains in the sample regardless of future price movement.<sup>2</sup> I replace any missing returns or prices with delisting returns and prices when possible.

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<sup>2</sup> If I instead require a stock to have a price greater than five dollars at the end of the prior month, all my results are unchanged.

I require stocks to have values for certain characteristics to enable asset pricing tests. Stocks must have a value for book-to-market (book value is from Compustat), market value, and beta at the beginning of month  $t$  to be part of the month  $t$  portfolio. I follow the construction methods of Fama-French (1992) to create these variables. In addition, a stock must have twelve months of prior returns.

For most tests, I use only stocks with a market capitalization greater than the 10% NYSE breakpoint.<sup>3</sup> I apply this constraint because microcap stocks have a disproportionate number of firms compared to their total market value, tend to cluster into the tails of characteristic sorts, and are often very illiquid. Including them in regressions would weight results towards an economically small portion of the market, and including them in portfolios (even value-weighted) would lead to trading patterns that would be expensive to execute.

I measure return volatility for each stock using a traditional risk approach:

$$\sigma_t^2 = \beta^2 \sigma_m^2 + \sigma_i^2$$

where  $\sigma_t^2$  is the total variance of returns,  $\beta$  is the CAPM beta, and  $\sigma_m^2$  is the variance of the market. For both  $\beta$  and  $\sigma_m^2$ , I follow Fama and French (1993) and define the market as the value-weighted CRSP sample of ordinary common shares trading on the NYSE, NASDAQ, or AMEX.  $\beta^2 \sigma_m^2$  is the systematic variance of returns.  $\sigma_i^2$  is the idiosyncratic variance and is calculated as  $\sigma_t^2 - \beta^2 \sigma_m^2$ .

I measure volatility using daily returns over intervals from one month to five years and monthly returns over intervals from six months to 10 years. Measurement lengths are

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<sup>3</sup> I thank Ken French for making the NYSE size breakpoints, the Fama-French four factor variables, and the Fama-French 49 industries specifications available on his website.

always matched when calculating values, e.g., total variance, market variance, and beta calculated from monthly returns over five years are used to calculate idiosyncratic volatility from monthly returns over five years. Stocks do not need to have the full time period of returns to enter the sample. For example, if only two years of returns are available then only two years of returns are used, even if the measurement period is five years.

The literature typically follows Ang, Hodrick, Xing, and Xiang (2006) in calculating idiosyncratic volatility. The idiosyncratic values I calculate closely match results generated from their method, especially over short intervals. For instance, using a one month interval, the two measures of idiosyncratic volatility have a correlation of about 0.99. Replacing my measure with their measure does not significantly change any results in the paper.

## **Results**

### *The characteristics and returns of the low volatility anomaly*

The difference in return between low and high volatility stocks is very large. Figure 1.1 shows the changing value of \$1.00 invested at the start of July 1980 in equal weighted portfolios formed on volatility. The low (high) volatility portfolio buys the stocks with the lowest (highest) 20% of idiosyncratic volatility measured over the past month using daily returns.<sup>4</sup> The stocks are resorted and the portfolios are rebalanced at the end of each month. At the end of 2011, the low volatility portfolio is worth \$89.50. The high volatility portfolio is worth only \$4.84. The performance of the low volatility portfolio is very similar to the

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<sup>4</sup> Other volatility measures produce similar results for this figure.

portfolio of the next 20% of volatility (\$86.28) and the 20% after that (\$85.94), but the high volatility portfolio is unique in its low return.

While Figure 1.1 shows only the unadjusted cumulative return for each portfolio, I would expect the low volatility portfolio to have lower measures of risk and therefore superior risk-adjusted returns as well. Table 1.1 presents common performance measures for each of the portfolios used in Figure 1.1. All measures are calculated from monthly returns, but I annualize the values presented. As expected, I find the low volatility portfolio has a lower standard deviation of returns (12.8% vs. 29.7%) and a higher arithmetic average return (10.2% vs. 4.4%). The variance of the high volatility portfolio creates a gap of about 4.4% between its arithmetic and geometric average return, but the gap is only about 0.8% for the low volatility portfolio. The low (high) volatility portfolio has an annual one-factor alpha of 5.6% (-5.9%) and a Fama-French four-factor alpha of 3.2% (-1.6%). The Sharpe and Treynor ratios of the low volatility portfolio are about 5x those of the high volatility portfolio. The low volatility portfolio has superior performance compared to the high volatility portfolio regardless of the evaluation technique.

The portfolios used are explicitly sorted on volatility, but there is much implicit sorting occurring within the quintiles. Table 1.2 shows a number of stock level characteristics for each volatility quintile. The values I present are the average of the median value for each quintile at the beginning of each month, except for the last five variables (Alpha through UMD). Those variables are the Fama-French four-factor results from the same regressions that produced the four-factor alphas in Table 1.1.

Since I explicitly sort on idiosyncratic volatility, some obvious results occur. Low idiosyncratic stocks also have a lower total and systematic standard deviation of monthly

returns, along with a lower beta. There are other less obvious, large differences between the groups though. The typical low volatility stock (\$1.54 billion dollars) is about 5x larger in market capitalization than the typical high volatility stock (\$306 million dollars). Stocks in the low volatility portfolio also have a higher book-to-market value (.63 vs. .46). These differences result in the high volatility portfolio having a large, positive SMB loading (1.15) and a negative HML loading (-.20). The low volatility portfolio has a much smaller SMB loading (.14) and a positive HML loading (.42).

The difference in alpha between the high and low volatility portfolios implies that differences in size and book-to-market do not explain the anomaly; however, the strength of the anomaly may vary within different groups.<sup>5</sup> I test the anomaly across size and book-to-market by dividing up the full sample (including stocks below the 10% NYSE size breakpoint) into different groups. First, all stocks below the 10% breakpoint are sorted into terciles by size, and all stocks above the 10% breakpoint are placed into one of three groups (between the 10% to 40% breakpoint, 40% to 70%, 70%+). Then within those groups, stocks are sorted into quintiles based on their book-to-market value. Finally, stocks within each size/book-to-market group are sorted into quintiles by idiosyncratic volatility measured over the past month using daily returns.

Table 1.3 shows the same measures from Table 1.1 for these size and book-to-market groups. I only present results from the highest (value) and lowest (growth) quintiles of book-to-market. The values shown are that of the low volatility quintile less that of the

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<sup>5</sup> I also calculated the average characteristic-based benchmark adjusted returns following Daniel, Grinblatt, Titman, and Wermers (DGTW) (1997) and Wermers (2003) for each quintile to ensure misspecification of the HML and SMB factors did not drive the result. The low volatility portfolio's average benchmark adjusted return exceeded that of the high volatility portfolio by about 1.1% per month. Russ Wermers makes the DGTW benchmarks available on his website.

high volatility quintile. Looking first at stocks above the 10% breakpoint, growth stocks have a larger anomaly by any measure. Among the 10% to 40% size group, growth stocks produce an anomaly with a geometric return (four-factor alpha) of 24.6% (16.0%) per year. The return (four-factor alpha) is only 10.6% (8.6%) for value stocks in the same size group. The anomaly is also weaker among larger stocks. The four-factor alpha for stocks above the 70% breakpoint is only 0.2% for value stocks (1.5% for growth).

I find the results are less clear among microcap stocks. The performance of the top two thirds of the microcaps by size is similar to that of the 10% to 40% group. The alphas and returns are large regardless of book-to-market. However, among the smallest third of microcaps I find the anomaly begins to flip. High volatility value stocks in the lowest size tercile outperform the matching low volatility stocks by an average of 12.4% per year, but the Sharpe and Treynor ratios for the low volatility stocks are larger than for the high volatility stocks. The results of this tercile are of questionable significance for portfolio formation purposes though. For instance, the combined market cap of the low volatility, growth component of this size group is only about \$446 million at the end of 2011. In comparison, ExxonMobil alone was worth about \$380 billion at that time.

Overall, it does appear the strength of the low volatility anomaly varies in the cross-section of size and book-to-market. The anomaly is stronger as size and book-to-market decrease, but not among the smallest of stocks. Among the large, value stocks the anomaly is weak by many measures.

*What form of volatility causes the anomaly?*

The low volatility anomaly literature is split between focusing on idiosyncratic volatility and focusing on total volatility. But, as shown Table 1.2, it can be difficult to study one without simultaneously studying the other. There is also little theoretical basis for the choice of measurement period. One month of daily returns is the most common period for idiosyncratic volatility, but Fu (2009) points out that is a very noisy measure of future idiosyncratic volatility.

Table 1.4 presents Fama-French four-factor alphas for the low and high volatility portfolios constructed as in Table 1.1, but with the volatility type, measurement period, and data granularity varied. Regardless of whether total volatility or idiosyncratic volatility is used, the difference in alpha between the low and high volatility quintile is large when volatility is measured over short intervals. Using one month of daily returns to measure idiosyncratic (total) volatility produces a difference in alpha between the low and high volatility portfolios of 4.8% (5.8%) per year. As the time period used to measure volatility increases to multiple years, the difference in alpha becomes statistically insignificant whether monthly or daily returns are used; although, the difference is still about 2.0% to 3.0% per year in those specifications. Neither beta nor systematic volatility is able to produce economically and statistically significant differences in alpha between the low and high volatility portfolios.

Table 1.4 indicates that shorter measurement periods produce a larger low volatility anomaly, but the results are not helpful in separating total and idiosyncratic volatility. I now take advantage of my construction of the volatility measures to separate their effects. Total volatility is the sum of systematic and idiosyncratic volatility. If only idiosyncratic

volatility affects stock returns, then systematic volatility should not affect returns. On the other hand, if total volatility matters, then both idiosyncratic volatility and systematic volatility should affect returns.

I test the effect of the different volatility forms using a panel of monthly stock returns. In a similar spirit to the Fama-Macbeth (1973) methods, I regress monthly stock returns on stock level characteristics; however, instead of running cross-sectional regressions and averaging coefficients, I use the full panel and include time and industry fixed effects.<sup>6</sup> The full model is:

$$r_{i,t+1} = \ln(\text{Size})_{i,t} + \text{BM}_{i,t} + \text{Mom}_{i,t} + \text{Beta}_{i,t} + \text{Vol}_{i,t} + \text{Time FE} + \text{Industry FE} + \varepsilon_{i,t}$$

where  $r_{i,t+1}$  is the percentage stock return for stock  $i$  in month  $t+1$ .  $\ln(\text{Size})_{i,t}$ ,  $\text{BM}_{i,t}$ ,  $\text{Beta}_{i,t}$  are market value, book-to-market, and beta for the stock measured as in Fama-French (1992).<sup>7</sup>  $\text{Mom}_{i,t}$  is the stock return over the previous twelve months.  $\text{Vol}_{i,t}$  represents any number of potential volatility measures.

I modify the variables from their original form for use in the regression. First, I winsorize all variables (including return) at the 99.5% and .5% levels to control for outliers.<sup>8</sup> I then z-score the right-hand side by month, i.e., each variable is less the mean of that variable that month and divided by the standard deviation of that variable that month. This controls for large differences in cross-sectional dispersion among the variables from month to month, particularly among volatility measures. It also gives the coefficients simple interpretations. Each coefficient can be read as the change in return from being one

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<sup>6</sup> If I specify the model using the Fama-Macbeth approach, the results are the qualitatively the same.

<sup>7</sup> If I use the Daniel, Grinblatt, Titman, and Wermers (DGTW) (1997) and Wermers (2003) characteristic-based benchmark adjusted returns instead of, or in addition to, stock level controls like size and book-to-market, the results are qualitatively the same.

<sup>8</sup> The conclusions drawn from the results are the same if returns are not winsorized.



standard deviation above the mean of that variable in the prior month. I exempt beta from winsoring and z-scoring since the Fama and French (1992) beta estimation method already controls for outliers, and because beta already has a straightforward interpretation.

I present results from this model in Table 1.5. Model (1) shows the method produces similar outcomes as the Fama-Macbeth style regressions used in other papers, e.g., Fama and French (2008). Size and beta have no statistically significant effect on returns, and returns are increasing in book-to-market and past return.

I add a measure of idiosyncratic and systematic volatility to the regression in Model (2). Being one standard deviation above the mean idiosyncratic volatility measured over the past six months using monthly returns decreases return in following month by 0.30% ( $p$ -value  $< 0.001$ ). Being one standard deviation above the mean in the matching systematic volatility decreases return in following month by only 0.04% ( $p$ -value = 0.523). I use daily returns over the past month to measure the volatilities instead in Model (3) and find quantitatively similar results. The combination of a strong effect of idiosyncratic volatility and no effect of systematic volatility implies that idiosyncratic, not total, volatility drives the low volatility anomaly. Extending the time period of measurement and mixing and matching measurements, i.e., sixty months of systematic volatility using monthly returns and one month of idiosyncratic volatility using daily returns, does not change the result.

I combine the variables of Models (2) and (3) in Model (4) and further find that idiosyncratic volatility does not appear to have a dominant form. Systematic volatility using either measure still has no effect, but both forms of idiosyncratic volatility are predictive of future returns. Being one standard deviation above the mean in the six (one) month measure of idiosyncratic volatility decreases returns in the following month by

0.18% (0.32%). Neither appears to completely capture the effect of the low volatility anomaly by itself.

Combining the two different idiosyncratic measures produces a larger anomaly than any single measure alone. Table 1.6 shows results from Fama-French four-factor regressions on equal weighted portfolios created from independently sorting stocks into quintiles based on two measures of idiosyncratic volatility: the past six months using monthly returns and the past month using daily returns. The measures are positively correlated (0.55), so there are a large numbers of stocks in the bottom or top quintile of both measures, but few in the bottom quintile of one measure and the top of the other. The stocks are resorted and the portfolios rebalanced at the end of each month. I only present results for portfolios formed from the bottom quintile of both measures (Low, Low) and the top quintile of both measures (High, High).

Over the full time period, the low idiosyncratic volatility portfolio has a positive alpha of about 0.33% per month. The high volatility counterpart has a negative alpha of about -0.33% per year. This 7.8% per year difference in risk-adjusted performance is about 2.0% greater than any single sort result in Table 1.4. By combining the two measures, the size of the low volatility anomaly increases significantly.

However, if the sample is split into two equal time periods, the anomaly only appears present in the first half of the sample.<sup>9</sup> Using only the returns from July 1980 through March 1996, the difference in alpha between the low and high volatility portfolios is about 1.4% per month. From April 1996 through December 2011, this difference drops to a statistically insignificant 0.14% per month ( $p$ -value = 0.660). The performance of the

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<sup>9</sup> This result also exists among the single volatility sorts.

low volatility portfolio does not change across periods (0.36% per month to 0.32%), but the high volatility portfolio increases its performance from -1.05% to 0.18% per month.

*What happened to the low volatility anomaly?*

The lack of a large risk-adjusted return during the second half of the sample is surprising because low volatility stocks still outperform high volatility stocks over that time. Figure 1.2 replicates Figure 1.1, but I begin the investment in April 1996 instead of July 1980. The high volatility portfolio still has far worse performance than all other quintiles. One dollar invested in the low (high) volatility quintile is worth \$5.74 (\$2.36) at the end of 2011. A low volatility portfolio would be expected to outperform a high volatility on a risk-adjusted basis if it outperformed in raw return.

I find the small difference in alpha between the low and high volatility portfolios in the second half of the sample is explained by the constraints of the factor model. The Fama-French four-factor model, like many others, generally assumes that factor exposures are constant over the sample period. Because high volatility stocks are small, high beta, growth stocks regardless of time period, the cost of this assumption is low for factors related to those risks; however, the assumption is costly with respect to momentum.

Figure 1.3 shows the momentum (UMD) coefficient for the high volatility portfolio used in Table 1.6 if the Fama-French four-factor regression is ran separately each year from 1997 through 2011. If the cost of assuming constant momentum exposure is low, the line should be approximately flat. Instead, the high volatility portfolio's momentum exposure experiences large swings above and below 0. The full sample has a UMD loading of about -0.60, but it drops as low as -1.34 in 2002 and moves as high as 0.35 in 2005.

The UMD exposure may change year to year in part because of its nonlinear relationship with the returns on high volatility stocks. To show this non-linearity, I rank the full sample of monthly returns of the high volatility portfolio, the CRSP value weighted index, and the UMD factor into one hundred groups. The highest (lowest) returns are placed into group 100 (1). Table 1.7 shows the returns for the three highest groups of high volatility portfolio returns and the returns and rankings for the CRSP index and the UMD factor in those same months.

Large returns to the high volatility portfolio are positively correlated with the overall market. The lowest ranking for the market returns among the largest high volatility returns is 66. The average ranking is 87. However, the trend is highly non-linear with respect to the UMD factor. Among the largest high volatility returns, the highest UMD ranking is 100, the lowest is 1, and none fall between 10 and 98. This nonlinearity is of particular importance in the second half of the sample. No returns to the high volatility portfolio before 1999 fall within the top 3 groups (top 11 returns).

The high volatility portfolio experiences large nonlinear UMD exposure during the second half of the sample because of market bubbles that began and ended over that time, e.g., the “dot-com” bubble and 2008 financial crisis. During a boom, high volatility stocks follow the market, perform very well, and enter the past winners section of the UMD measure. After a crash, high volatility stocks follow the market, perform very poorly, and enter the past losers section of the UMD measure. The UMD exposure of high volatility stocks is then driven by which side of the bubble the market sits. An example of this trend can be seen in Figure 1.3 from 1998 through 2001. The UMD exposure of the high volatility portfolio increased monotonically from -0.71 in 1998 to 0.2 in 2000, the height of the “dot-

com” bubble; however, the UMD exposure experienced a large drop to -1.20 in 2001 after the market crashed.

To account for this issue, I re-measure the alpha of the high and low volatility portfolios using a new specification for momentum. First, I follow the same steps to construct the UMD factor used in the standard Fama-French four-factor model, but use only stocks eligible for my volatility portfolios. This change focuses momentum, like my stock sample, on the most economically relevant portion of the stock market.<sup>10</sup> Second, I include the squared value of this new momentum measure in the model to account for non-linearity.<sup>11</sup>

Table 1.8 shows the results with this new momentum factor for the second half of the sample. First, there is little effect on the low volatility portfolio. Its momentum exposure is small regardless of model. Alpha changes by less than 1 basis point per month when UMD is reformed using only portfolio eligible stocks. It increases by 0.1% per month when a squared measure is included. On the other hand, the high volatility portfolio results have a large change when momentum is modified. The linear momentum exposure remains negative (-0.26), but there is a large positive non-linear effect (0.58). Alpha drops from 0.18% per month to -0.08% month when UMD is reformed using only portfolio eligible stocks and further decreases to -0.56% per month when a squared measure is included. The difference in alpha between high and low volatility portfolio moves from 0.14% per month with the original UMD factor, to 0.39% per month with the modified UMD factor, and finally to 0.98% per month (about 11.8% per year) with the squared UMD factor.

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<sup>10</sup> The implications of the upcoming results are unchanged if UMD itself is not modified.

<sup>11</sup> I also used a momentum variable equal to UMD if the momentum return is greater than some cut-off point, e.g., 0%. It produced similar results. The result is not sensitive to the cut-off point chosen.

These results indicate that the low volatility anomaly has not disappeared over the last 15 years. It is the misspecification of the momentum factor that makes it appear that the anomaly is gone. The presence of market bubbles makes adapting the momentum measure particularly important. During non-bubble periods, the misspecification is less damaging to inference. If the results from Table 1.8 are repeated for the first half of the sample, the alpha of the high volatility portfolio decreases by only 8 basis points per month and the squared momentum term is statistically insignificant.

## **Conclusions**

I study the low volatility anomaly and find it produces large raw and risk-adjusted returns from 1980 through 2011. One dollar invested in an equal weighted portfolio of low volatility stocks in July 1980 is worth \$89.50 at the end of 2011. The matching portfolio of high volatility stocks is worth only \$4.84. The Sharpe and Treynor ratios of the same low volatility portfolio are about 5x those of the matching high volatility portfolio. I find evidence of the low volatility anomaly at all levels of market capitalization, except among the smallest stocks, and note that the anomaly is stronger among growth stocks than value stocks.

I show that portfolios formed on either total or idiosyncratic volatility will produce large risk-adjusted returns, but my panel regressions indicate that idiosyncratic volatility is the primary driver of the low volatility anomaly. Being one standard deviation above the mean idiosyncratic volatility measured over the past six months decreases return in following month by .30%. However, I do not find that there is a dominant form of idiosyncratic volatility that explains the full anomaly. Instead, I show that a low volatility

portfolio based on two different measures of idiosyncratic volatility (six months of monthly returns and one month of daily returns) produces a Fama-French four-factor alpha about 7.8% per year greater than that of a matching high volatility portfolio.

Finally, I find that the anomaly appears to disappear from 1996 to 2011 when measured using the Fama-French four-factor model. I attribute this absence to model misspecification resulting from the relationship between momentum and high volatility stocks. High volatility stocks move quickly between being past winners and past losers as market bubbles form and collapse. Because this occurs multiple times from 1996 to 2011, the UMD exposure of the high volatility portfolio experiences significant changes over time. After adjusting UMD for my stock sample and adding a squared term, the difference in alpha between the low and high volatility stock portfolios from 1996 to 2011 changes from about 1.7% per year to about 11.8% per year.

**Table 1.1: The Risk-Adjusted Return to the Low Volatility Anomaly**

This table shows various measures of return for five equal weighted portfolios formed on volatility. The low (high) volatility portfolio buys the 20% of stocks in the sample with the lowest (highest) idiosyncratic standard deviation of returns over the past month using daily returns. I define the idiosyncratic standard deviation of returns as the square root of the difference between the total and systematic variances. The performance is measured from July 1980 through December 2011. The stocks are resorted and the portfolios rebalanced at the end of each month. All measurements are made with the returns less the risk-free rate. Standard deviation is the annualized standard deviation of the monthly portfolios returns. The arithmetic return is the annualized average monthly return for the portfolio. Geometric return is the annualized geometric return for the portfolio. 1-Factor Alpha is the annualized CAPM alpha of the portfolio. 4-Factor Alpha is the annualized Fama-French four-factor alpha of the portfolio. The Sharpe ratio is the arithmetic return divided by the standard deviation. The Treynor ratio is the arithmetic return divided by the CAPM beta of the portfolio.

SD Group	Standard Deviation	Arithmetic Return	Geometric Return	1-Factor Alpha	4-Factor Alpha	Sharpe Ratio	Treynor Ratio
Low	12.8%	10.2%	9.4%	5.6%	3.2%	0.80	0.15
2	16.1%	10.6%	9.3%	4.6%	2.4%	0.66	0.12
3	18.6%	11.0%	9.3%	4.0%	2.6%	0.59	0.10
4	22.1%	10.6%	8.2%	2.3%	2.9%	0.48	0.08
High	29.7%	4.4%	0.0%	-5.9%	-1.6%	0.15	0.03
L - H	-16.9%	5.8%	9.4%	11.5%	4.8%	0.65	0.12



**Table 1.2: The Characteristics of Low and High Volatility Stocks**

This table shows the characteristics of low and high volatility stocks. The low (high) volatility group is the 20% of stocks in the sample with the lowest (highest) idiosyncratic standard deviation of returns over the past month using daily returns. I define the idiosyncratic standard deviation of returns as the square root of the difference between the total and systematic variances. Stocks are sorted each month from July 1980 through December 2011. The median value for each characteristic in each group is then calculated at the beginning of each month and the average of the median values is presented, except for Alpha, FF beta, SMB, HML, and UMD. Those variables are the factor loadings from regressing the portfolios returns from Table 1.1 against the Fama-French four-factor model. SD is the total standard deviation of daily returns over the past month. Idio SD (Syst SD) is the idiosyncratic (systematic) portion of the standard deviation of returns. Beta is the CAPM beta of the stock over the past month. Market Value is market capitalization (size) of the stock. B-to-M is the book-to-market value for the stock.

	Low	2	3	4	High	L - H
SD	1.2%	1.7%	2.2%	2.8%	4.1%	-3.0%
Idio SD	1.0%	1.4%	1.9%	2.5%	3.7%	-2.8%
Syst SD	0.5%	0.7%	0.9%	1.0%	1.3%	-0.8%
Beta	0.54	0.75	0.88	1.02	1.22	-0.68
Market Value	1540	950	612	436	306	1234
B-to-M	0.63	0.58	0.56	0.51	0.46	0.16
Alpha	0.27%	0.20%	0.22%	0.24%	-0.13%	0.40%
FF Beta	0.77	0.96	1.06	1.13	1.27	-0.49
SMB	0.14	0.28	0.47	0.73	1.15	-1.00
HML	0.42	0.40	0.31	0.07	-0.20	0.62
UMD	0.00	-0.03	-0.08	-0.18	-0.49	0.49

**Table 1.3: Do Low Volatility Stocks Outperform High Volatility Stocks Regardless of Size and Book-to-Market?**

This table shows the difference in performance between low and high volatility stocks among different size and book-to-market groups. Stocks are first sorted into one of six size groups. Stocks below the 10% NYSE size breakpoint are sorted into size terciles. Stocks above the 10% NYSE breakpoint are sorted into one of three groups: between the 10% to 40% NYSE breakpoint, between the 40% to 70% breakpoint, and above the 70% breakpoint. Stocks within each size groups are then sorted into quintiles based on their book-to-market value. Finally, stocks are sorted into quintiles by their idiosyncratic standard deviation of returns over the past month using daily returns. I define the idiosyncratic standard deviation of returns as the square root of the difference between the total and systematic variances. The stocks are sorted each month, and the portfolios are equal weighted. I only present results for the lowest 20% (growth) and highest 20% (value) of book-to-market. Each value presented in the table represents the low volatility result less than high volatility result for that group. All performance measures are defined the same as in Table 1.1.

Size Group	BM Group	Standard Deviation	Arithmetic Return	Geometric Return	1-Factor Alpha	4-Factor Alpha	Sharpe Ratio	Treynor Ratio
Bottom 33% of	Growth	-24.1%	-1.8%	6.0%	1.8%	-2.7%	0.16	0.04
Bottom 10%	Value	-24.0%	-12.4%	-5.1%	-8.8%	-12.7%	0.09	0.03
Middle 33% of	Growth	-18.0%	10.5%	15.6%	13.6%	8.1%	0.39	0.10
Bottom 10%	Value	-13.4%	22.4%	25.5%	25.2%	22.9%	1.19	0.32
Top 33% of	Growth	-25.0%	17.1%	24.7%	21.5%	13.6%	0.56	0.14
Bottom 10%	Value	-16.0%	9.1%	13.0%	12.8%	11.3%	0.73	0.18
10%-40%	Growth	-17.8%	19.8%	24.6%	24.3%	16.0%	0.80	0.16
	Value	-15.0%	7.3%	10.6%	11.5%	8.6%	0.64	0.13
40%-70%	Growth	-18.5%	8.7%	13.4%	13.9%	2.6%	0.51	0.10
	Value	-13.9%	-1.1%	1.8%	3.5%	0.9%	0.33	0.07
70%+	Growth	-18.0%	4.3%	8.7%	10.0%	1.5%	0.49	0.09
	Value	-14.6%	-2.2%	0.9%	3.1%	0.2%	0.18	0.06

**Table 1.4: How Sensitive Is the Low Volatility Anomaly to the Method of Measurement?**

This table shows the monthly Fama-French four-factor alpha for portfolios of low and high volatility stocks over the time period July 1980 through December 2011. The low (high) volatility portfolio is an equal weighted portfolio of stocks with the lowest (highest) 20% of past volatility as measured by total standard deviation of past returns, CAPM beta, systematic standard deviation of past returns, or idiosyncratic standard deviation of past returns. The measures of volatility are calculated using either daily or monthly returns over time intervals ranging from one month to ten years. The stocks are sorted and the portfolios rebalanced at the end of each month. Robust standard errors are used to test if the alphas of the low and high volatility portfolios are different from zero and if the alphas of the low and high volatility portfolios are different from one another. \*, \*\*, and \*\*\* represent statistical significance at the 10%, 5%, and 1% levels.

Return Frequency	# of Months	Total Volatility			Beta			Systematic Volatility			Idiosyncratic Volatility		
		Low	High	Diff	Low	High	Diff	Low	High	Diff	Low	High	Diff
Daily	1	0.0029***	-0.0019	0.0048**	0.0014	-0.0002	0.0016	0.0019**	-0.0005	0.0024	0.0027***	-0.0013	0.0040**
	3	0.0029***	-0.0015	0.0044**	0.0014	0.0009	0.0005	0.0015	0.0007	0.0007	0.0029***	-0.0016	0.0045**
	6	0.0029***	-0.0011	0.0040*	0.0019**	0.0011	0.0008	0.0020**	0.0010	0.0010	0.0029***	-0.0011	0.0040*
	12	0.0031***	-0.0003	0.0034	0.0023**	0.0015	0.0008	0.0023**	0.0015	0.0008	0.0029***	-0.0005	0.0034*
	36	0.0029***	0.0001	0.0028	0.0024***	0.0020	0.0003	0.0024***	0.0020	0.0004	0.0028***	0.0001	0.0027
	60	0.0026***	0.0002	0.0025	0.0023***	0.0018	0.0005	0.0023***	0.0018	0.0005	0.0024***	0.0001	0.0023
Monthly	6	0.0027***	-0.0009	0.0036**	-0.0005	0.0012	-0.0017	0.0007	0.0007	-0.0000	0.0033***	-0.0009	0.0042***
	12	0.0027***	0.0002	0.0025	-0.0005	0.0018	-0.0022	0.0002	0.0016	-0.0014	0.0031***	-0.0002	0.0033**
	36	0.0026***	-0.0001	0.0026	0.0006	0.0024	-0.0018	0.0008	0.0023	-0.0016	0.0027***	-0.0003	0.0029
	60	0.0026***	0.0001	0.0024	0.0008	0.0023	-0.0016	0.0009	0.0021	-0.0012	0.0025***	-0.0002	0.0027
	120	0.0021***	0.0005	0.0016	0.0011	0.0027*	-0.0016	0.0009	0.0024*	-0.0015	0.0020***	0.0003	0.0018

**Table 1.5: What Form of Volatility Creates the Low Volatility Anomaly?**

This table presents results from the following model:

$$r_{i,t+1} = \ln(\text{Size})_{i,t} + \text{BM}_{i,t} + \text{Mom}_{i,t} + \text{Beta}_{i,t} + \text{Vol}_{i,t} + \text{Time FE} + \text{Industry FE} + \varepsilon_{i,t}$$

The dependent variable is the stock return of stock  $i$  in month  $t+1$ . I include up to four  $\text{Vol}_{i,t}$  variables. Idio 6 Months is the idiosyncratic standard deviation of returns measured over the previous six months using monthly returns. Idio 1 Month is the idiosyncratic standard deviation of returns measured over the previous month using daily returns. Syst 6 Months and Syst 1 Month are the systematic standard deviation of returns over the same time periods using the same data.  $\ln(\text{Size})$  is the natural log of a market capitalization, B-to-M is the book-to-market value, and Beta is the CAPM beta of the stock. All are constructed following Fama and French (1992). Momentum is the return on the stock over the previous twelve months. I include year-month fixed effects and industry fixed effects. I specify industry by the Fama-French 49 industries specification first proposed in Fama and French (1997). The time period is July 1980 through December 2011. All variables, except beta, are winsorized at the .5% and 99.5% levels. All right-hand side variables, except beta, are z-scored within their year-month of measurement.  $p$ -values calculated from standard errors clustered on time are reported below the coefficients in brackets. \*, \*\*, and \*\*\* represent statistical significance at the 10%, 5%, and 1% levels.

	(1)	(2)	(3)	(4)
$\ln(\text{Size})$	0.00 [0.963]	-0.06 [0.291]	-0.09 [0.109]	-0.11* [0.053]
B-to-M	0.19*** [0.000]	0.18*** [0.000]	0.18*** [0.000]	0.18*** [0.000]
Momentum	0.23** [0.023]	0.25** [0.016]	0.21** [0.042]	0.22** [0.031]
Beta	-0.27 [0.291]	-0.03 [0.879]	0.01 [0.958]	0.07 [0.693]
Idio 6 Months		-0.30*** [0.000]		-0.18*** [0.000]
Idio 1 Month			-0.39*** [0.000]	-0.32*** [0.000]
Syst 6 Months		-0.04 [0.523]		0.00 [0.947]
Syst 1 Month			-0.02 [0.748]	-0.01 [0.850]
Observations	738,938	738,938	738,938	738,938
Number of Months	378	378	378	378

**Table 1.6: How Strong Is the Low Volatility Anomaly?**

This table shows Fama-French four-factor regression results for portfolios of low and high volatility stocks over the time period July 1980 through December 2011. The Low, Low (High, High) volatility portfolio is an equal weighted portfolio of stocks which are in the lowest (highest) quintile of idiosyncratic volatility as measured by both the last six months using monthly returns and the last month using daily returns. The stocks are sorted and the portfolios rebalanced at the end of each month. I divide the sample into equal sections by time and test the portfolios from July 1980 through March 1996 in Models (4) through (6) and from April 1996 through December 2011 in Models (7) through (9). *p*-values from robust standard errors are reported below the coefficients in brackets. \*, \*\*, and \*\*\* represent statistical significance using robust standard errors at the 10%, 5%, and 1% levels.

	(1) Low, Low	(2) High, High	(3) (1) - (2)	(4) Low, Low	(5) High, High	(6) (4) - (5)	(7) Low, Low	(8) High, High	(9) (7) - (8)
Beta	0.7186*** [0.000]	1.3367*** [0.000]	-0.6181*** [0.000]	0.7743*** [0.000]	1.2156*** [0.000]	-0.4412*** [0.000]	0.6553*** [0.000]	1.3907*** [0.000]	-0.7355*** [0.000]
SMB	0.0757** [0.027]	1.3085*** [0.000]	-1.2328*** [0.000]	0.1162** [0.033]	1.0146*** [0.000]	-0.8983*** [0.000]	0.0886** [0.012]	1.4254*** [0.000]	-1.3368*** [0.000]
HML	0.4040*** [0.000]	-0.2723*** [0.000]	0.6763*** [0.000]	0.3087*** [0.000]	-0.1769** [0.044]	0.4856*** [0.000]	0.4640*** [0.000]	-0.3081*** [0.000]	0.7721*** [0.000]
UMD	0.0132 [0.537]	-0.5259*** [0.000]	0.5391*** [0.000]	-0.0015 [0.969]	-0.2005*** [0.001]	0.1990** [0.027]	-0.0024 [0.919]	-0.6025*** [0.000]	0.6001*** [0.000]
Alpha	0.0033*** [0.000]	-0.0033* [0.069]	0.0065*** [0.004]	0.0036*** [0.001]	-0.0105*** [0.000]	0.0141*** [0.000]	0.0032*** [0.008]	0.0018 [0.443]	0.0014 [0.660]
Observations	378	378	378	189	189	189	189	189	189
Adjusted $r^2$	0.821	0.906	0.772	0.849	0.897	0.635	0.823	0.931	0.844
Time Period	Full	Full	Full	07/80-03/96	07/80-03/96	07/80-03/96	04/96-12/11	04/96-12/11	04/96-12/11

**Table 1.7: How Does the High Volatility Portfolio Respond to the Market and Momentum?**

This table shows how the market and the momentum factor performed during months when the high volatility portfolio had its highest returns. The high volatility portfolio is an equal weighted portfolio of stocks which are in the both the highest quintile of idiosyncratic volatility using the last six months of monthly returns and the last month using daily returns. All returns from July 1980 through December 2011 for the high volatility portfolio, the CRSP value-weighted index, and UMD are ranked. I then sort each into 100 groups based on those rankings. The highest (lowest) returns are placed into group 100 (1). I show the returns for the top three groups of high volatility portfolio returns and the corresponding market and UMD returns and rankings for those same months.

Date	High Volatility Portfolio		Market Return		UMD Return	
	Return	Rank	Return	Rank	Return	Rank
May-97	19.4%	98	7.2%	93	-5.2%	9
Apr-03	19.5%	98	8.3%	97	-9.4%	2
Oct-01	20.7%	98	2.8%	66	-8.4%	4
Jun-00	20.8%	98	5.2%	86	16.6%	100
Dec-99	22.1%	99	8.4%	97	13.2%	99
May-03	24.6%	99	6.4%	90	-10.8%	2
Mar-09	29.3%	99	8.8%	98	-11.5%	2
Nov-02	33.8%	99	6.1%	89	-16.3%	1
Feb-00	36.3%	100	3.2%	70	18.4%	100
Jan-01	41.8%	100	4.0%	75	-25.0%	1
Apr-09	50.9%	100	11.1%	99	-34.8%	1

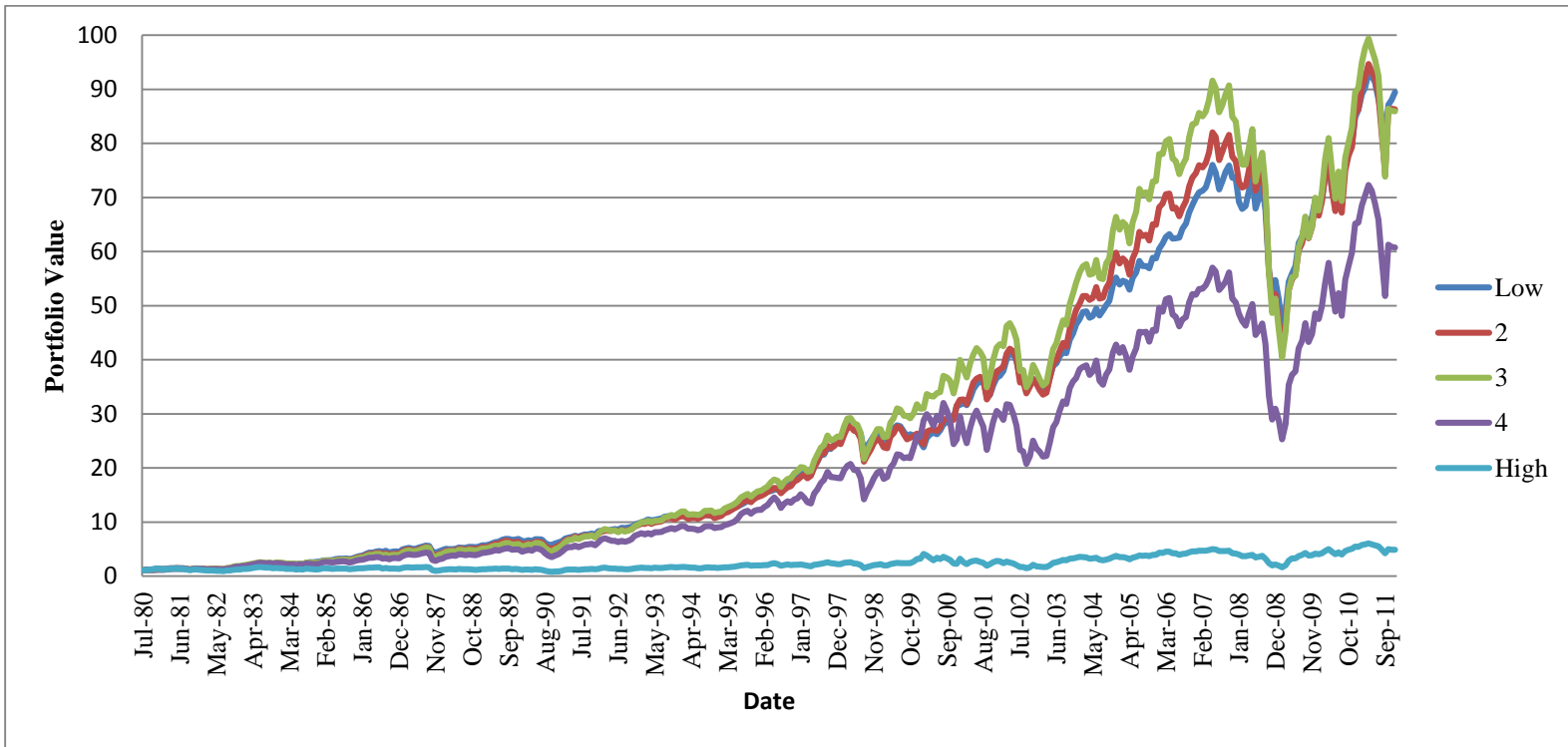
**Table 1.8: How Does the Specification of Momentum Affect the Strength of the Low Volatility Anomaly?**

This table shows the Fama-French four-factor regression results for portfolios of low and high volatility stocks over the time period April 1996 through December 2011. The Low, Low (High, High) volatility portfolio is an equal weighted portfolio of stocks which are in the lowest (highest) quintile of idiosyncratic volatility by both the last six months using monthly returns and the last month using daily returns. The stocks are sorted and the portfolios rebalanced at the end of each month. New UMD is constructed in the same manner as the original UMD factor, but using only stocks that meet the volatility portfolio criteria. New UMD<sup>2</sup> is the squared value of New UMD factor. *p*-values calculated from robust standard errors are reported below the coefficients in brackets. \*, \*\*, and \*\*\* represent statistical significance at the 10%, 5%, and 1% levels.

	(1) Low, Low	(2) Low, Low	(3) Low, Low	(4) High, High	(5) High, High	(6) High, High	(7) (1) - (4)	(8) (2) - (5)	(9) (3) - (6)
Beta	0.6553*** [0.000]	0.6570*** [0.000]	0.6557*** [0.000]	1.3907*** [0.000]	1.4440*** [0.000]	1.4504*** [0.000]	-0.7355*** [0.000]	-0.7870*** [0.000]	-0.7947*** [0.000]
SMB	0.0886** [0.012]	0.0881** [0.015]	0.1150*** [0.001]	1.4254*** [0.000]	1.3235*** [0.000]	1.1894*** [0.000]	-1.3368*** [0.000]	-1.2354*** [0.000]	-1.0745*** [0.000]
HML	0.4640*** [0.000]	0.4656*** [0.000]	0.4593*** [0.000]	-0.3081*** [0.000]	-0.3539*** [0.000]	-0.3227*** [0.000]	0.7721*** [0.000]	0.8195*** [0.000]	0.7820*** [0.000]
UMD	-0.0024 [0.919]			-0.6025*** [0.000]			0.6001*** [0.000]		
New UMD		0.0013 [0.922]	-0.0219 [0.207]		-0.3785*** [0.000]	-0.2626*** [0.000]		0.3798*** [0.000]	0.2407*** [0.000]
New UMD <sup>2</sup>			-0.1162** [0.014]			0.5799*** [0.000]			-0.6961*** [0.000]
Alpha	0.0032*** [0.008]	0.0032*** [0.008]	0.0042*** [0.001]	0.0018 [0.443]	-0.0008 [0.732]	-0.0056*** [0.005]	0.0014 [0.660]	0.0039 [0.177]	0.0098*** [0.000]
Observations	189	189	189	189	189	189	189	189	189
Adjusted r <sup>2</sup>	0.821	0.827	0.833	0.906	0.937	0.952	0.772	0.855	0.885

### Figure 1.1: The Return to the Low Volatility Anomaly – July 1980 through December 2011

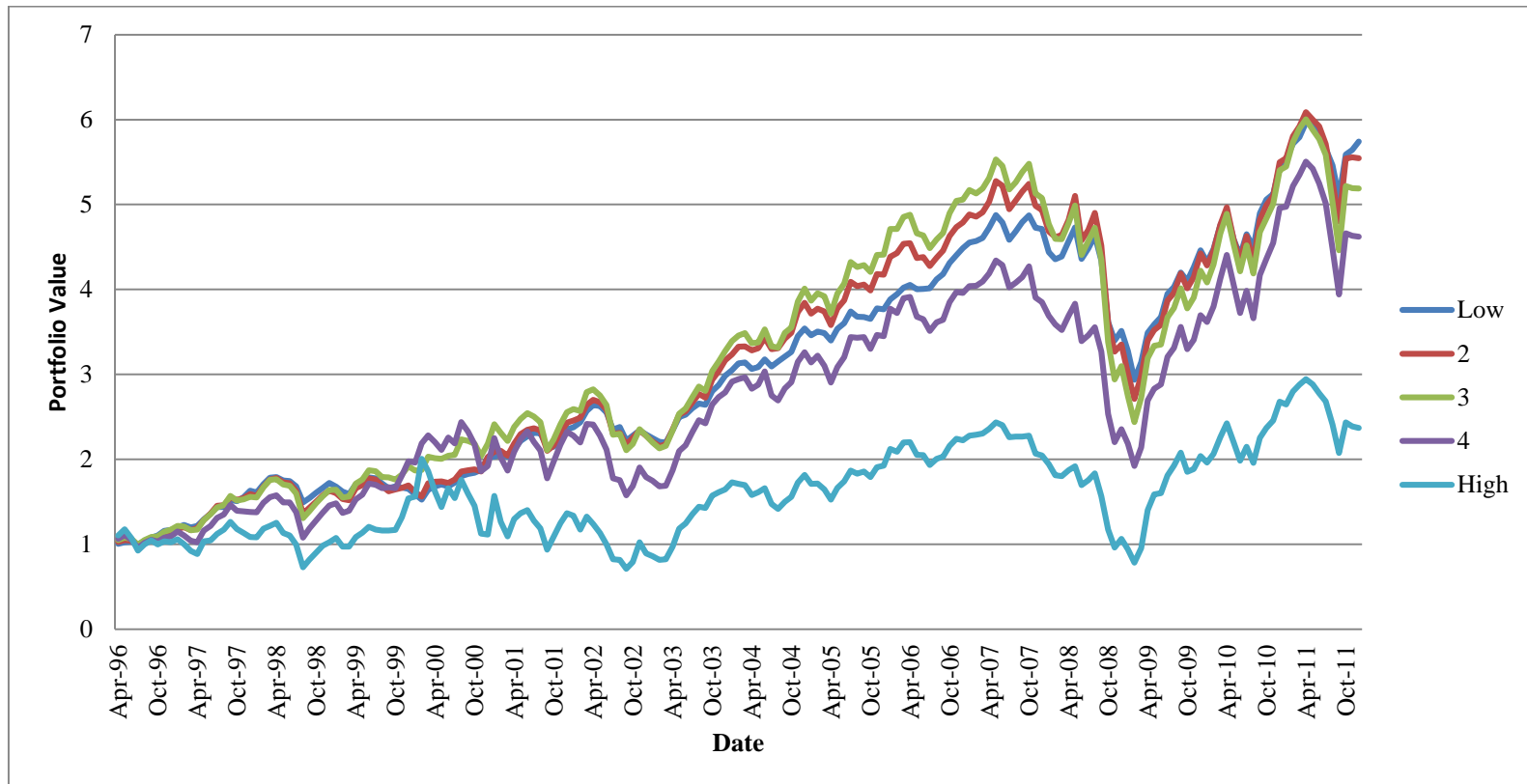
This figure shows the changing value of \$1.00 invested from July 1980 through December 2011 in five equal weighted portfolios formed on volatility. The low (high) volatility portfolio buys the 20% of stocks in the sample with the lowest (highest) idiosyncratic standard deviation of returns over the past month using daily returns. I define the idiosyncratic standard deviation of returns as the square root of the difference between the total and systematic variances. The stocks are sorted and the portfolios rebalanced at the end of each month.





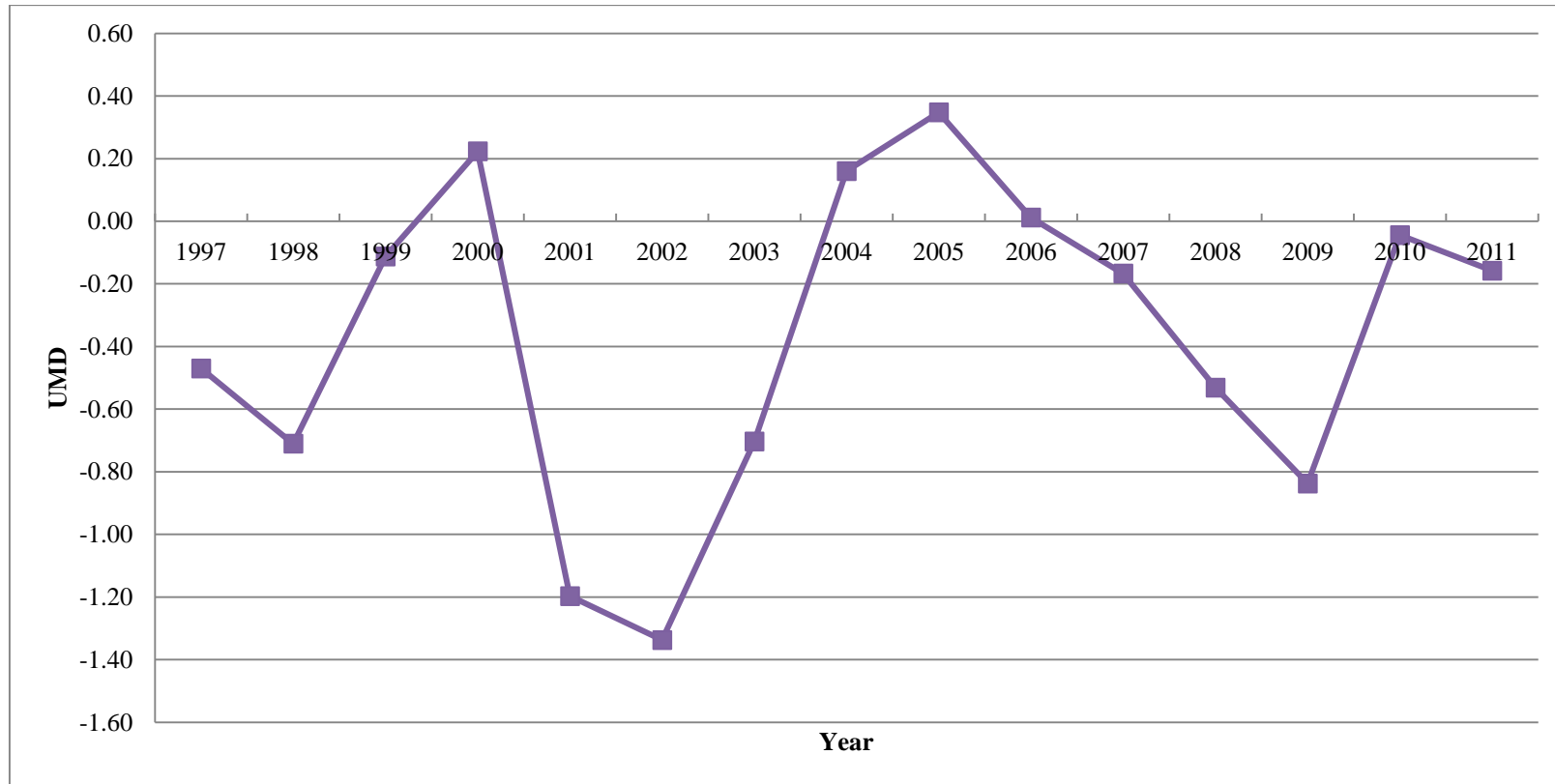
### Figure 1.2: The Return to the Low Volatility Anomaly – April 1996 through December 2011

This figure shows the changing value of \$1.00 invested from April 1996 through December 2011 in five equal weighted portfolios formed on volatility. The low (high) volatility portfolio buys the 20% of stocks in the sample with the lowest (highest) idiosyncratic standard deviation of returns over the past month using daily returns. I define the idiosyncratic standard deviation of returns as the square root of the difference between the total and systematic variances. The stocks are sorted and the portfolios rebalanced at the end of each month.



**Figure 1.3: How Does the High Volatility Portfolio's Momentum Exposure Change Over Time?**

This figure shows the year-by-year momentum exposure, i.e., UMD factor exposure, of the high volatility portfolio used in Table 1.6. The Fama-French four-factor regression is performed separately each year from 1997 through 2011.



## **Chapter 2: The Low Volatility Anomaly and Mutual Fund Manager Skill**

### **Introduction**

It is well understood that past mutual fund performance is not necessarily indicative of future results. Whether a manager's performance will persist is difficult to determine from past returns alone because the process of attributing those returns to luck or skill is an uncertain process. As the efficiency of a market increases, the problem becomes more difficult. There is less opportunity to outperform the market and separating the signal from the noise is harder. In a reasonably efficient market, any simple procedure designed to pick skilled managers based on past returns alone would not be expected to be successful.

Despite that expectation, I show that the past return volatility of active equity mutual funds is a powerful determinant of future performance. One dollar invested in a portfolio of mutual funds with low past return volatility at the beginning of 2000 is worth about \$2.00 at the end of 2011. A portfolio of mutual funds with high past return volatility invested over the same time period has an ending value of only \$0.73. In comparison, a zero-fee fund tracking the CRSP value weighted index starting in January 2000 would be worth \$1.19 at the end of 2011.

I find that a portfolio of low volatility funds has a Fama-French four-factor alpha about 5.4% per year greater than that of a portfolio of high volatility funds. This difference in performance between low and high volatility funds is robust to changes in evaluation models, changes in the sample of funds, and controlling for fund characteristics. Low volatility mutual funds do tend to be larger, older, and have lower expenses and turnover, but these differences do not explain their performance. My panel regressions show that a

one standard deviation decrease in fund volatility in the prior year predicts an increase in the alpha of a fund of about 2.5% in the following year.

These initial results support the notion that managers of funds with low return volatility have more skill than high volatility fund managers; however, that statement requires the assumption that the pricing model is correct. An alternative explanation is that the difference in performance is driven by systematic bias in the model. I argue that the difference in performance is not skill, but instead results from models not taking into account the well-documented low volatility anomaly.

The low volatility anomaly states that returns are decreasing as past return volatility increases. Baker, Bradley, and Wurgler (2011) find that from January 1968 through December 2008 the geometric return to a portfolio of low volatility stocks is over 11.0% per year greater than the matching portfolio of high volatility stocks. This difference occurs despite the low volatility portfolio having a standard deviation of returns that is less than half that of the high volatility portfolio. It is possible that fund managers could have generated strong returns with low risk by using a mechanical stock picking rule based on volatility alone.

I find a similar performance gap between low and high volatility funds exists among both funds that deviate from their benchmark and those funds that do not. If the difference in performance is skill, and not the low volatility anomaly, it should not exist among funds that are “closet indexers”. I then simulate mutual funds that invest in either low or high volatility stocks and find returns similar to the performance of actual low and high volatility mutual funds. The average return for an equal weighted portfolio of 50 low volatility stocks

was about 12.3% per year greater than the average return to an equal weighted portfolio of 50 high volatility stocks.

I next add a new pricing factor, LVmHV (low volatility minus high volatility), to the Fama-French four-factor model to remove the mechanical reward for holding low volatility stocks. The difference in alpha between low and high volatility mutual funds drops to only 0.84% per year with the addition of LVmHV. After further accounting for differences in liquidity, the difference in alpha further decreases to about 0.12% per year. I compare the distribution of low and high volatility fund alphas to simulated distributions where fund managers are constrained to have no skill in tests similar to those of Kosowski, Timmerman, Wermers, and White (2006) and Fama and French (2010). After accounting for the low volatility anomaly, I find no evidence of skill among low volatility fund managers and no evidence of a difference in skill between low and high volatility fund managers.

My results suggest that current tests of manager skill are biased by not controlling for the clear reward for holding low volatility stocks. Managers who invest in small stocks or value stocks have their leanings taken into account when evaluating their performance. By failing to take into account volatility in the same way, low volatility managers can appear skilled and high volatility managers can appear unskilled regardless of their true ability. By accounting for the low volatility anomaly, inferences on mutual fund manager skill will be greatly improved.

## Literature Review

Conventional wisdom says that mutual fund managers cannot consistently beat the market, i.e., their skill does not justify their expense. Sharpe (1966) and Jensen (1968) first showed that the average mutual fund underperformed given its level of risk. Sharpe (1991) further demonstrates that the average dollar invested in mutual funds must have a negative alpha because investing is (1) a zero sum game and (2) funds have expenses. He refers to this point as “the arithmetic of active management.” Carhart (1997) attributes any persistence in equity mutual fund performance to the momentum effect, not manager skill.<sup>12</sup> Fama and French (2010) use the distribution of mutual fund alphas to demonstrate that few funds generate long run risk-adjusted returns sufficient to offset fund expenses. While most work has focused on equity funds, a lack of manager skill has also been found among bond funds (Chen, Ferson, and Peters (2010)) and among general institutional money managers (Busse, Goyal, and Wahal (2010)).

Counter to the conventional wisdom, there is a large body of research that finds fund managers do have skill. Daniel, Grinblatt, Titman, and Wermers (1997) find that fund managers are able to select stocks that outperform their peers. Wermers (2000) finds this same skill, but shows that non-stock holdings, fund expenses, and transaction costs more than offset any stock picking ability. The stocks selected by equity mutual funds outperform the market by about 1.3% per year, but fund net returns underperform by about 1.0% per year. Chen, Jegadeesh, and Wermers (2000) find that the stocks most commonly held by mutual funds do not outperform the market, but that the stocks bought by funds do

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<sup>12</sup> Many papers had documented persistence in returns prior to this paper. For instance: Brown and Goetzmann (1995), Elton, Gruber, and Blake (1996), Goetzmann and Ibbotson (1994), Grinblatt and Titman (1992), and Hendricks, Patel, and Zeckhauser (1993).

outperform stocks sold by funds. Baker, Litov, Wachter, and Wurgler (2010) focus on fund trades made around earnings announcements and find managers on average anticipate earnings surprises correctly

Much of the selection skill among mutual fund managers is related to industry selection rather than individual security selection. Kacperczyk, Sialm, and Zheng (2005) find that managers who deviate from a well-diversified portfolio and concentrate their holdings in specific industries perform better. Busse and Tong (2005) show that funds with industry selection skill have more persistent alpha. Avramov and Wermers (2006) find that predictability in future fund returns is best identified using measures of manager industry selectivity.

While Fama and French (2010) attribute most positive alphas to luck, not skill, similar research finds top performing funds do have persistent performance. Kosowski, Timmerman, Wermers, and White (2006) also study the distribution of fund alphas and find that many funds generate consistent, positive alphas not attributable to luck. They find that skilled funds add about \$1.2 billion per year in wealth to the mutual fund industry, although funds that appear to lack skill remove about \$1.5 billion per year in wealth. Barras, Scaillet, and Wermers (2010) find that about 75% of funds have a positive stock-picking history inconsistent with luck, but extract rent through expenses instead of leaving investors with positive alpha.

Bayesian techniques have also found evidence for managerial skill. Avramov and Wermers (2006) show that a performance evaluator who believes in fund manager skill will select funds with stronger future performance than one who does not believe in skill. Busse and Irvine (2006) likewise find that evaluating net of fee performance while allowing

for the possibility of managerial skill increases the ability to identify strong future performers. Huij and Verbeek (2007) use a Bayesian estimation approach to demonstrate that mutual fund performance does persist in the short run.

The ability to find skill is often dependent on the type of data and method used. Bollen and Busse (2001) find that mutual funds can time the market, but that the effect is only strong in daily data. Bollen and Busse (2005) also use daily data to show that mutual fund returns persist, but only over short-intervals. This short persistence makes the frequency of performance measurement and portfolio sorting important. Kothari and Warner (2001) find that typical performance measures, such as the Fama-French four-factor model, have low power in detecting abnormal performance. They advocate using event studies procedures focused on fund trading to improve the power of performance evaluation.

Furthermore, heterogeneity in fund manager skill can be captured by measuring differences between fund holdings. Busse, Green, and Baks (2006) show that mutual funds that take large positions on a small number of stocks outperform widely diversified funds. Cremers and Petajisto (2009) create a measure of stock selectivity called Active Share and find that funds whose holdings deviate more (less) from their benchmark tend to outperform (underperform) their benchmark. Cremers, Ferreira, Matos, and Starks (2013) find that result holds among mutual funds worldwide. Amihud and Goyenko (2013) avoid holdings and create a measure of stock selectivity using the  $r^2$  resulting from the regression of fund returns on stock pricing models. They find results similar to those generated by holdings data. More selective, i.e., lower  $r^2$ , funds have better returns, especially among top past performers.



Fund manager skill can also be identified by the characteristics of fund managers. Golec (1996) first studied how managerial characteristics affected fund performance and found young managers with MBAs had the strongest performance. Chevalier and Ellison (1999) study the same topic and find that many of correlations between managerial characteristics and fund performance arise from differences in risk between funds. The only robust relationship they identify is that managers with degrees from higher-SAT colleges have higher returns. Whether the difference in return arises because those managers are naturally brighter, better educated, have better networks, or are hired into better companies is unclear.

There has been little work linking the low volatility anomaly in stocks to mutual fund manager skill. Qin (2013) finds that short-term persistence of skill among mutual funds can be attributed to funds holding high idiosyncratic stocks (as defined by the Fu (2009) GARCH measure). He calculates an idiosyncratic volatility pricing factor and finds it captures much of the difference in short term future performance between funds with low and high past returns.

## **Data and Methods**

I use the CRSP Survivor-Bias-Free U.S. Mutual Fund database to build my sample of actively managed U.S. equity funds. I drop any funds that (1) CRSP identifies as index funds, ETFs, or variable annuities, (2) have a Lipper asset code of TX or MB, or (3) have terms in their name not associated with active management or equity investment.<sup>13</sup> I also

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<sup>13</sup> I drop funds with any permutation of following terms in their fund name: bond, cash, convertible, cycle, ETF, fixed, government, index, ishare, lifestyle, maturity, money, mortgage, municipal, powershare, principal protection, profund, proshare, rate, real estate, realty, tax, term, treasury, variable, 2005, 2010, 2015, 2020, 2025, 2030, 2035, 2040, 2045, 2050, 2055, 529.

require that a fund have at least 80% of its assets invested in equity during the previous year and have a Lipper class code consistent with equity investing.<sup>14,15</sup> To control for the incubation bias of Evans (2010), I restrict the sample to funds that are at least a year old and have at least \$20 million in assets.

I combine multiple share classes of a single fund using the CRSP class group variable (*crsp\_cl\_grp*). The assets of the combined fund are the sum of the assets held across all share classes. I weight all other fund attributes (including return) by the assets held in each share class.

I use the daily return file to calculate measures of past performance and volatility for each fund each calendar year. The file begins in September 1998, so I first measure results in 1999. I calculate the fund return, the total and idiosyncratic standard deviation of returns, and alpha and factor loadings from the Fama-French four-factor model for each fund that records a return every day during each calendar year. I use the standard deviation of the residuals from the Fama-French four-factor regression as my idiosyncratic standard deviation of returns.

## **Results**

### *The performance of low and high volatility mutual funds*

I first capture the difference in performance between low and high volatility mutual funds by sorting funds into portfolios based on past return volatility. At the beginning of

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<sup>14</sup> CRSP is missing this information from 1998 through 2002 for most funds, so I check this constraint using asset allocations from 1997 to determine the sample in 1999 through 2003.

<sup>15</sup> I use funds with the following Lipper class codes associated with market cap and value/growth tilt: EIEI, LCCE, LCGE, LCVE, MCCE, MCGE, MCVE, MLCE, MLGE, MLVE, SCCE, SCGE, SCVE. If I expand the list of eligible codes to include funds that use other strategies, e.g., LSE – Long/Short Equity, my results are unchanged.

each year, funds are sorted into deciles based on the standard deviation of their daily fund returns during the prior calendar year. The low (high) volatility portfolio holds the 10% of mutual funds in the sample with the lowest (highest) standard deviation of daily returns in the prior calendar year. Each portfolio is equal weighted and has the same number of funds at the start of the year. A fund remains in the same portfolio for the entire year.

Figure 2.1 shows the value of one dollar invested in the five such volatility portfolios starting in January 2000 and ending December 2011. The portfolio holding funds with the lowest volatility of past returns outperformed all the others. The low volatility portfolio is worth about \$2.00 at the end of 2011. The high volatility portfolio is worth only \$0.73. The trend holds within the middle groups as well, e.g., funds in the third lowest volatility group beat those in the fifth and seventh, and those in the fifth lowest group beat those in the seventh.

Table 2.1, Panel A shows the average return and performance evaluation measures for those same portfolios. The arithmetic (geometric) average return for the low volatility portfolio was 6.0% (8.4%) greater per year than that of the high volatility portfolio. That large difference occurs with the low (high) volatility portfolio having an annualized standard deviation of returns of 14.2% (25.8%). The Sharpe and Treynor ratios of the high volatility portfolio are both slightly negative while the low volatility portfolio has the highest ratios among all ten portfolios. The low volatility portfolio has the best performance regardless of the method of evaluation.

Table 2.1, Panel B shows the correlation between the monthly returns of the volatility portfolios. The returns of all the portfolios are positively related, but I find the relationship grows weaker as the difference in volatility increases. The low volatility

portfolio and the third lowest volatility portfolio have a return correlation of 0.98. The high volatility portfolio and the seventh lowest volatility portfolio have a return correlation of 0.96. However the low and high volatility portfolios have a return correlation of only 0.83. That correlation is still high in absolute terms, but it is small relative to the strength of the relationship between the other portfolios.

While the difference in return between the low and high volatility funds is large, it is possible that well-known market anomalies could explain the result. Table 2.2 shows the Fama-French four-factor alpha and exposures for the low and high volatility portfolios. The low volatility portfolio outperforms the market by about .16% per month (1.9% per year) and outperforms the high volatility portfolio by about .45% per month (5.4% per year). Low volatility funds do tend to hold low beta, value stocks, and high volatility funds do tend to hold high beta, small cap, growth stocks; however, these differences do not explain the difference in performance. If the portfolios are formed in January but only evaluated in January through June or July through December the results are similar. That result indicates that it is not necessary to update measurements of fund volatility often to maintain the difference in performance.

While the overall difference in performance is large, the low volatility portfolio does not outperform the high volatility portfolio every year. Figure 2.2 shows the difference in monthly percentage alpha between the low volatility fund portfolio and the high volatility fund portfolio for each year of the sample. The high volatility funds outperformed the low volatility funds by about 0.50% per month in 2000 and by smaller margins in 2007 and 2010. The low volatility funds had their strongest performance in 2002 in beating the high volatility portfolio by 1.2% per month. Low volatility funds also

exceeded the performance of the high volatility funds by over 0.50% per month in 2001, 2004, and 2008. The overall performance of the low volatility funds is robust to the removal of these best years though. If years 2001, 2002, 2004, and 2008 are dropped from the sample, the overall difference in alpha between low and high volatility funds is still about 0.31% per month ( $p$ -value = 0.085).

The result that low volatility funds outperform high volatility funds after controlling for anomalous returns is also robust to the choice of evaluation model. Table 2.3, Panel A tests the difference in alpha between the low and high volatility fund portfolios using alternative evaluation models. I again start with the Fama-French four-factor model, but then add the liquidity factors of Pastor and Stambaugh (2003) and Sadka (2006).<sup>16</sup> I then substitute the Cremers, Petajisto, and Zitzewitz (2012) seven-factor (CPZ7) model for the Fama-French model and repeat the tests.<sup>17</sup> In addition, I present results for the original equal weighted portfolios and for the same portfolios weighted by fund assets to test if small funds alone are driving the result.

I find that the difference in performance between the low and high volatility portfolios is large regardless of the model.<sup>18</sup> The equal weighted portfolio evaluated with the CPZ7 model and all the liquidity factors has an alpha of about 0.19% per month ( $p$ -value = 0.169). The asset weighted portfolio with the same model and factors has an alpha of about 0.25% per month ( $p$ -value = 0.063). These results indicate that allowing for effect of liquidity and modifying the factor model does lower alpha, but still leaves a large gap in performance between high and low volatility funds. Since the effect is as strong for the

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<sup>16</sup> I thank Ronnie Sadka for making his liquidity factors available at his website.

<sup>17</sup> I thank the authors for making their pricing factors available at their website.

<sup>18</sup> The base result in Model (1) varies slightly from Table 2.2 because 2011 is excluded. Neither the CPZ7 nor the Sadka factors are available for that year.

asset weighted portfolios as it is for the equal weighted portfolios, it does not appear that small funds are driving my results.

I further test the robustness of the original result by excluding certain types of funds in Table 2.3, Panel B. I test the difference in alpha using the Fama-French four-factor model, but I now exclude certain groups from the sample before sorting the funds into the portfolios. In particular, I exclude funds with less than \$300 million in assets and funds with a small cap or growth orientation.<sup>19</sup> The asset limit further tests if small funds drive the result and the orientation exclusions test if the result is driven by those risky fund types alone. If a particular group is included (excluded) in the sample, the table marks the category row Yes (No).

Excluding any of the groups lowers alpha by about 0.10% to 0.20% per month. Excluding all the groups simultaneously decreases the equal weighted alpha from 0.45% per month to 0.28% per month ( $p$ -value = .008). In untabulated results, that same portfolio had an equal weighted alpha of about 0.13% per month ( $p$ -value = 0.227) and value weighted alpha of about 0.15% per year ( $p$ -value = 0.191) using the CPZ7 model with all liquidity factors included. Taken as a whole, the results in Table 2.3 suggest that fund size, risk orientation, and the evaluation model together explain some, but not all, of the difference in performance between low and high volatility funds.

The full difference in performance may instead be related to heterogeneity in other fund characteristics. Table 2.4 shows the characteristics of funds when they first enter the low and high volatility portfolios. Most surprising is that funds sorted into the high volatility portfolio have an average return about 2.8% greater than those in the low

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<sup>19</sup> Small cap funds are identified by Lipper classes SCGE, SCVE, SCCE. Growth funds are identified by Lipper classes LCGE, MLGE, MCGE, and SCGE.

volatility portfolio; however, the median return for funds entering the low volatility portfolio is about 1.7% greater than that of those funds entering the high volatility portfolio. Low (high) volatility funds have an average four-factor alpha of 0.07% (-1.14%) per year, so on average the sorting process does place funds with high (low) past risk-adjusted into the low (high) volatility portfolio.

Low volatility funds have different levels of systematic and unsystematic risk compared to high volatility funds. Low volatility funds have about half the daily standard deviation of returns of high volatility funds (0.98% against 1.82%), and Panel B shows that low volatility funds have lower market risk, less small cap exposure, less growth exposure, and less momentum exposure than high volatility funds. The difference in total volatility of returns is driven in part by these different preferences for systematic risks, but high volatility funds also have about twice the daily idiosyncratic volatility as low volatility funds (0.51% against 0.28%).

The average (median) size for the low volatility funds is \$3.0 billion (\$360 million), but only \$901 million (\$240 million) for the high volatility funds. The average low volatility fund is about 4 years older than the average high volatility fund, charges 0.18% less in expenses per year, and has a turnover of only 56.6% (116.6% for high volatility funds). The low expense and low turnover of the low volatility funds are indicators of future strong performance, e.g., Carhart (1997), but large fund size has been found to lower returns, e.g., Chen, Hong, Huang, and Kubik (2004).

I test if past volatility or other fund characteristics predict future performance using the following panel model:

$$\text{Alpha}_{i,t+1} = \text{Alpha}_{i,t} + \text{SD}_{i,t} + \text{Idio}_{i,t} + \text{Fund Controls}_{i,t} + \text{Obj FE} + \text{Time FE} + \epsilon_{i,t}$$

where  $\text{Alpha}_{i,t+1}$  is the annualized (250 day) percentage alpha for fund  $i$  in calendar year  $t+1$  calculated from the Fama-French four-factor model using daily returns.  $\text{Alpha}_{i,t}$  is the same alpha in the prior year.  $\text{SD}_{i,t}$  and  $\text{Idio}_{i,t}$  are the standard deviation and idiosyncratic standard deviation of the daily returns in year  $t$ . Fund Controls $_{i,t}$  include the natural log of fund assets, natural log of age, expense, and turnover all as of December of year  $t$ . I also include all four Fama-French four-factor exposures measured using daily returns during year  $t$ . Both Lipper objective (a course categorization) and Lipper class (a finer categorization) fixed effects are included in addition to year fixed effects.<sup>20</sup> I cluster the standard errors on year and calculate them using a bootstrap procedure. All continuous variables are winsorized at the .5% and 99.5% levels.<sup>21</sup> All continuous right-hand-side variables are z-scored (demeaned and divided by their standard deviation) so that the coefficients can be interpreted as the change in alpha from a one standard deviation change in the variable.<sup>22</sup>

I present results from this model in Table 2.5. The standard deviation of past returns is the strongest predictor of future fund performance. A one standard deviation increase in the standard deviation of past returns decreases alpha by about 2.5% in the next year. Idiosyncratic volatility is not predictive of future alpha, and past alpha is a weak predictor. Smaller funds, older funds, and funds with low turnover and expense have statistically significant greater alpha, but the effects are small (0.20% to 0.40%). The Fama-French exposures have no statistically significant effect when the standard deviation of past returns is included in the model.

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<sup>20</sup> The model can be adapted to a Fama-Macbeth (1973) specification without changing the implication of the results.

<sup>21</sup> The unwinsorized results generate the same conclusions.

<sup>22</sup> This normalization does not affect the statistical significance of the coefficients.



*Do low volatility mutual fund managers have skill?*

My results so far have indicated that fund managers with low volatility past returns outperform fund managers with high volatility past returns. Even after accounting for heterogeneity in fund characteristics, past volatility is a strong predictor of future performance. Recent work by Cremers and Petajisto (2009) and Amihud and Goyenko (2013) shows that managers who choose to deviate more from their benchmark or the overall market perform better than other managers. It may be the case that the difference in return between low and high volatility funds is caused by differences in stock selectivity skill between low and high volatility fund managers.

I test this possibility using two measure of stock selectivity: the Active Share measure of Cremers and Petajisto (2009) and the  $r^2$  measure of Amihud and Goyenko (2013). Active Share for a fund is equal to the sum of the absolute deviations between fund holdings and benchmark holdings.<sup>23</sup> A lower (higher) Active Share implies a less (more) selective manager.  $r^2$  for a fund is the  $r^2$  value calculated after regressing the past twenty four months of monthly returns against the Fama-French four-factor model. A lower (higher)  $r^2$  implies a more (less) selective manager.

I first sort funds into quintiles based on their Active Share or  $r^2$  each month. I use the most recent value for Active Share available unless that value is more than three months old. A fund whose most recent value of Active Share is more than three months old is ineligible for inclusion in the portfolios that month. For the  $r^2$  measure, I require a fund to have at least the past twelve months of returns to be eligible for inclusion in the portfolios that month. I then sort funds within the Active Share or  $r^2$  quintiles into quintiles based on

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<sup>23</sup> I thank Antii Petajisto for making the Active Shares measures from Petajisto (2013) available on his website.

the standard deviation of fund daily returns in the prior calendar year. My time period is reduced to January 2001 through December 2009 for this double sort because only during that period are both the  $r^2$  and Active Share measures available.

If stock selectivity is driving the result, then I should find no difference in performance between high and low volatility funds among funds with low selectivity. However, Table 2.6 shows that the difference in performance between low and high volatility funds exists regardless of the level of selectivity. For the Active Share results in Panel A, the overall difference in performance between the low and high volatility funds is about 0.36% per month. Among the least (most) selective funds this difference is about 0.21% (0.54%) per month. For the  $r^2$  results in Panel B, the overall difference in performance between the low and high volatility funds is about 0.38% per month. Among the least (most) selective funds this difference is about 0.33% (0.41%) per month. So while stock selectivity may affect fund performance, that ability does not appear to explain the large gap in performance driven by volatility.<sup>24</sup>

A second possibility is that low volatility mutual funds outperform high volatility mutual funds because of the low volatility anomaly. As discussed before, low volatility stocks have been shown to consistently outperform high volatility stocks. As a result, I would expect that mutual funds formed on the basis of stock volatility alone would have a large difference in performance. To test this possibility, I first simulate portfolios of low volatility stocks and portfolios of high volatility stocks operating over the same time period as my original mutual fund sample, January 2000 through December 2011. If the simulated low volatility portfolios outperform the simulated high volatility portfolios, then the low

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<sup>24</sup> Active Share and  $r^2$  have small, statistically insignificant effects if placed in the panel model of Table 2.5.

volatility anomaly may help explain the difference in performance between real low and high volatility mutual funds.

I use the CRSP stock files to build a specific sample of stocks. My goal is to focus on the most economically relevant portion of U.S. equities, i.e., those most commonly held by mutual funds. I use only ordinary shares (CRSP share codes 10 and 11) that trade on the NYSE, NASDAQ, or AMEX (CRSP exchange codes 1, 2, and 3). I consider a stock a penny stock and omit it from the sample until its price exceeds \$5 at the end of a month. From that point forward, it remains in the sample regardless of future price movement.<sup>25</sup> I only use stocks with a market capitalization greater than the 10% NYSE breakpoint to remove microcaps. I replace any missing returns or prices with delisting returns and prices when possible.

To create my sample of simulated low and high volatility mutual funds, I first sort all stocks that pass my screens into deciles at the beginning of every year based on the standard deviation of their monthly returns over the previous calendar year. I then randomly choose 50 stocks from the lowest decile of volatility and hold them in an equal, value, or randomly weighted portfolio. The same stocks remain in the portfolio for the full upcoming calendar year unless they fail a screen. At the beginning of each year, 50 new stocks are randomly chosen. Value weighted portfolios use the market capitalization of stocks to generate weights. Randomly weighted portfolios use the same market capitalization weights as the value weighted portfolios but randomly assign them to stocks. I follow this procedure to form 1,000 low volatility portfolios of each weighting. I then

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<sup>25</sup> If I instead require a stock to have a price greater than five dollars in the month prior to each sort my results are unchanged.

repeat this process using only stocks from the highest decile of volatility to form 1,000 high volatility portfolios of each weighting.

Table 2.7 shows the performance of the simulated low and high volatility funds. Like with the real low volatility funds, the simulated low volatility funds outperform the simulated high volatility funds regardless of the method of evaluation. Looking at the equal weighted results in Panel A, the arithmetic (geometric) average return of the simulated low volatility funds is 12.3% (21.5%) greater per year than that of the simulated high volatility funds. That large difference occurs with the low (high) volatility portfolio having an annualized standard deviation of returns of 12.5% (44.8%). The low volatility portfolio has large, positive Sharpe and Treynor ratios while the high volatility portfolio has small, negative ratios. The results are similar using value or random weighting.

To account for this mechanical difference in performance, I introduce a new pricing factor, LVmHV (low volatility minus high volatility), into the Fama-French four-factor model. I calculate the factor using the same sample and volatility measurement as in the simulated portfolios. Each month the LVmHV factor is equal to the return on a value weighted portfolio of all stocks that pass my screens that are in the lowest decile of standard deviation of monthly returns during the previous calendar year less the return to a value weighted portfolio of all stocks that pass my screens in the highest decile. This factor should control for any difference in performance between mutual funds that arises because of the low volatility anomaly.

Table 2.8, Panel A shows the basic characteristics of the LVmHV factor and compares it to the Fama-French four-factors and the Pastor and Stambaugh (2003) liquidity factor. The LVmHV factor has a mean (median) return of 1.16% (0.16%). Compared to

the Fama-French four-factors, LVmHV has a large mean return. LVmHV is also about two to three times as volatile as the four-factors. Panel B shows the correlation of LVmHV with the other pricing factors. While most pricing factors have a low correlation with one another, LVmHV is highly correlated with multiple factors. It has a correlation of -0.71 with the market risk factor (Mktrf), -0.66 with the market capitalization factor (SMB), and 0.52 with the value factor (HML). Given that low volatility stocks are typically low beta, large market capitalization, and high book-to-market compared high volatility stocks, these relationships are as expected.

I reproduce the results of Table 2.2 using LVmHV as an additional factor in Table 2.9. Alpha for the low volatility portfolio falls from 0.16% per month to 0.03% with the addition of LVmHV. Alpha for the high volatility portfolio increases from -0.30% per month to -0.05%. This moves the difference in alpha between the portfolios down from about 0.45% to about 0.07% per month. If I include the Pastor and Stambaugh (2003) liquidity factor, the difference in alpha drops to only 0.01% per month ( $p$ -value = 0.902). Based on these results, once the benefit of the low volatility anomaly is removed there is little difference in performance between high and low volatility mutual funds. Low volatility fund managers are not more skilled than high volatility fund managers; they are just benefiting from a well-established market anomaly not accounted for in the typical pricing models.

The relationship between stock volatility and low and high volatility mutual funds can be further seen in Table 2.10. For each volatility portfolio, I present the annualized standard deviation of monthly returns and two different measures of idiosyncratic volatility. The first (FF4) measures the idiosyncratic volatility as the annualized standard

deviation of the residuals from a Fama-French four-factor regression using monthly returns. The second (FF4 + LVmHV) is same except that LVmHV is included in the regression as a pricing factor.

The standard deviation of the portfolios increases monotonically as the volatility group increases, but idiosyncratic volatility has a U-shape. The low (high) volatility portfolio has an idiosyncratic volatility of 3.33% (5.52%) in the FF4 specification, but the sixth ranked portfolio has an idiosyncratic volatility of only 2.24%. When LVmHV is included as a pricing factor, this U-shape flattens significantly. Idiosyncratic volatility for the low (high) volatility portfolio drops 15.8% (23.1%) while the fifth ranked portfolio has a change of only -0.1%. The low and high volatility portfolios have large portions of their volatility that appear idiosyncratic, but the LVmHV factor shows that much of that apparent idiosyncratic volatility is actually systematically driven by the low volatility anomaly.

As a final demonstration of how the LVmHV factor effects measurements of low and high volatility alpha, I now look at the distribution of low and high volatility fund alphas. I compare the actual distributions of both low and high volatility fund alphas to a theoretical distribution where all alpha is due to luck, not skill. To create the zero-skill distribution I follow either the bootstrap procedure of Kosowski, Timmerman, Wermers, and White (KTWW) (2006) or that of Fama and French (2010). Among other computational differences, the KTWW method only uses funds with at least sixty months of returns while the Fama-French method use funds with at least eight months of returns. This difference creates a larger survivorship bias in the KTWW results. Both methods use

the  $t$ -statistic associated with alpha rather than alpha itself to control for different levels of risk-taking across funds.

In this test, I now consider a mutual fund low (high) volatility in every month after the standard deviation of its daily returns first falls into the lowest (highest) 10% among funds in the prior calendar year. Only fund months between January 2000 and December 2011 and after a fund is labeled low or high volatility are used in the analysis. For each figure that follows, I present (1) a plot of the cumulative distribution of the alpha of low volatility mutual funds, (2) a plot of the cumulative distribution of the alpha of high volatility mutual funds, and (3) a combined distribution of low and high volatility fund alpha calculated under the restriction that fund managers have no skill. The presented figures vary depending on whether or not LVmHV is included as a pricing factor, whether the KTTW or Fama-French bootstrap method is used, and whether gross or net returns are used. The analysis of net returns indicates whether fund managers have the ability to produce post-fee returns higher than would be expected by luck alone. The analysis of gross returns indicates whether fund managers have the ability to select stocks that performs better than would be expected by luck alone.

Figure 2.3 presents cumulative distribution using the Fama-French four-factor model without the LVmHV factor. If the cumulative distribution of  $t$ -statistics for the low or high volatility portfolio is lower (higher) than the no-skill distribution at a point on the figure, then better (worse) performance than would be expected by luck alone is occurring. Using the KTTW methods and net returns in Figure 2.3A, I find that a substantial number of low volatility funds appear to have positive post-fee performance not explainable by luck. I find no evidence that any high volatility funds have skill. The results using the

Fama-French methods in Figure 2.3B are similar, but weaker, for the low volatility funds. Using gross returns in Figures 2.3C and 2.3D, I find evidence that nearly all low volatility managers are able to select stocks that perform better than would be expected by luck alone. Some high volatility funds select stocks that perform about as well as would be expected by luck alone, but most do not.

Figure 2.4 presents the same figures as in Figure 2.3 but includes the LVmHV factor as a pricing factor along with the other Fama-French factors. After accounting for the low volatility anomaly, the performance of the low and high volatility funds is similar on a net or gross basis. Using net returns in Figures 2.4A and 2.4B, the low and high volatility funds have a similar distributions of alpha. Both distributions have worse performance than would be expected by luck alone except in the far tails. Using gross returns in Figure 2.4C and 2.4D, I find that both low and high volatility fund managers select stocks that perform about as well as would be expected by luck alone. In particular, using the Fama-French method in Figure 2.4D, the low volatility, high volatility, and no-skill distributions are nearly identical.

## **Conclusions**

I find that a fund's return volatility during the prior calendar year is a powerful determinant of future performance. A portfolio of low volatility funds has an alpha of 5.4% per year greater than a portfolio of high volatility funds. After controlling for heterogeneity in fund characteristics, I find a one standard deviation decrease in fund volatility in the prior year predicts an increase in the alpha of a fund of about 2.5% in the following year. However, I find that this difference in performance is not related to skill, but is a result of



the low volatility anomaly. That is, common evaluation models are systematically biased towards funds that hold low volatility stocks.

I simulate low and high volatility mutual funds based on stock volatility and find returns similar to returns on actual low and high volatility mutual funds. The average return to an equal weighted portfolio of 50 low volatility stocks is about 12.3% per year greater than the average return to an equal weighted portfolio of 50 high volatility stocks. I create a new pricing factor, LVmHV (low volatility minus high volatility), to control for the low volatility anomaly and find it reduces the difference in alpha between real low and high volatility mutual funds to only 0.84% per year. Further, I perform bootstrapped alpha tests that show no evidence of (1) skill among low volatility funds or (2) difference in skill between low and high volatility funds after including the LVmHV factor in the pricing model. Overall, my results suggest that accounting for the low volatility anomaly is an essential part of properly specifying tests of manager skill because a large systematic bias in favor of low volatility funds exists in common evaluation models.

**Table 2.1: The Returns on Portfolios of Mutual Funds Sorted on Past Volatility**

This table shows the return on five equal weighted portfolios of active U.S. equity mutual funds. The low (high) volatility portfolio buys the 10% of mutual funds in the sample with the lowest (highest) standard deviation of daily returns in the prior calendar year. I only present the 1<sup>st</sup> (low volatility), 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup>, and 10<sup>th</sup> (high volatility) deciles. Panel A shows the performance of the each portfolio from January 2000 through December 2011. Average Return is the mean monthly return for the portfolio multiplied by twelve. Geometric Return is the monthly compound return for the portfolio compounded over twelve months. Standard Deviation is the annualized standard deviation of monthly portfolio returns. Sharpe (Treyner) Ratio is the average of the monthly returns less the risk-free rate divided by the portfolio standard deviation (CAPM beta). Panel B shows the correlation of monthly returns across the portfolios.

Panel A: Portfolio Returns						
	Low	3	5	7	High	L - H
Average Return	6.8%	5.0%	3.5%	2.4%	0.8%	6.0%
Geometric Return	5.9%	3.7%	2.1%	0.6%	-2.5%	8.4%
Standard Deviation	14.2%	16.0%	16.9%	18.8%	25.8%	-11.6%
Sharpe Ratio	0.32	0.17	0.07	0.01	-0.06	0.38
Treynor Ratio	0.06	0.03	0.01	0.00	-0.01	0.07

Panel B: Portfolio Return Correlations					
	Low	3	5	7	High
Low	1				
3	0.98	1			
5	0.97	0.98	1		
7	0.93	0.93	0.98	1	
High	0.83	0.81	0.89	0.96	1

**Table 2.2: Do Low Volatility Mutual Funds Outperform High Volatility Mutual Funds?**

This table shows the Fama-French four-factor regression results for monthly returns on portfolios of low and high volatility mutual funds from January 2000 through December 2011. The low (high) volatility portfolio is an equal weighted portfolio of active U.S. equity funds with the lowest (highest) 10% of the standard deviation of daily returns in the prior calendar year. I divide the sample into equal sections and test the portfolios only in the first six months of the year in Models (4) through (6) and only in the last six months in Models (7) through (9). *p*-values from robust standard errors are reported below the coefficients in brackets. \*, \*\*, and \*\*\* represent statistical significance at the 10%, 5%, and 1% levels.

	Full Sample			January - June			July - December		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Low	High	L - H	Low	High	L - H	Low	High	L - H
Beta	0.79*** [0.000]	1.24*** [0.000]	-0.45*** [0.000]	0.73*** [0.000]	1.31*** [0.000]	-0.58*** [0.000]	0.82*** [0.000]	1.21*** [0.000]	-0.39*** [0.000]
SMB	0.08*** [0.000]	0.54*** [0.000]	-0.46*** [0.000]	0.05 [0.118]	0.53*** [0.000]	-0.48*** [0.000]	0.17*** [0.000]	0.55*** [0.000]	-0.38*** [0.000]
HML	0.25*** [0.000]	-0.28*** [0.000]	0.54*** [0.000]	0.23*** [0.000]	-0.32*** [0.000]	0.55*** [0.000]	0.26*** [0.000]	-0.24*** [0.002]	0.50*** [0.000]
UMD	0.02 [0.356]	0.04 [0.479]	-0.02 [0.758]	0.01 [0.700]	0.03 [0.652]	-0.03 [0.756]	0.04 [0.233]	0.07 [0.190]	-0.03 [0.690]
Alpha	0.16%** [0.044]	-0.30%** [0.020]	0.45%*** [0.008]	0.15% [0.151]	-0.28% [0.106]	0.43%* [0.073]	0.18%* [0.074]	-0.32%** [0.041]	0.51%** [0.019]
Observations	144	144	144	72	72	72	72	72	72
Adjusted $r^2$	0.94	0.95	0.79	0.94	0.95	0.82	0.96	0.96	0.76

**Table 2.3: How Robust Is the Difference in Alpha Between Low and High Volatility Mutual Funds?**

This table shows the difference in monthly percentage alpha between the low and high deciles of mutual funds sorted by volatility. The low (high) volatility portfolio is a portfolio of active U.S. equity funds with the lowest (highest) 10% of the standard deviation of daily returns in the prior calendar year. Portfolios are weighted using either equal or total net asset (TNA) weighting. The difference in alpha is measured from January 2000 through December 2010 for Panel A and from January 2000 through December 2011 for Panel B. Panel A shows results using both the Fama-French four-factor model (FF4) and the Cremers, Petajisto, and Zitzewitz (2012) seven factor (CPZ7) model. I also include the Pastor and Stambaugh (2003) and Sadka (2006) liquidity factor in some specifications. Yes (No) indicates that the factor was (was not) included in the regression. Panel B shows results using the Fama-French four-factor model with certain groups of mutual funds excluded. In different specifications, I drop small funds (assets less than \$300 million at the beginning of the year), funds that primarily invest in small stocks (Lipper classes SCGE, SCVE, and SCCE), and funds that primarily invest in primarily in growth stocks (Lipper classes LCGE, MLGE, MCGE, and SCGE). Yes (No) indicates that the group was (was not) included in the sort. *p*-values from robust standard errors are reported below the coefficients in brackets.

Panel A: Different Models								
Factor Model	FF4	FF4	FF4	FF4	CPZ7	CPZ7	CPZ7	CPZ7
PS Liquidity	No	Yes	No	Yes	No	Yes	No	Yes
Sadka Liquidity	No	No	Yes	Yes	No	No	Yes	Yes
Equal Weight	0.42%** [0.024]	0.22% [0.212]	0.41%** [0.029]	0.23% [0.212]	0.30%* [0.050]	0.18% [0.175]	0.30%** [0.047]	0.19% [0.169]
TNA Weight	0.41%** [0.019]	0.26% [0.127]	0.41%** [0.020]	0.27% [0.121]	0.33%** [0.019]	0.24%* [0.068]	0.33%** [0.018]	0.25%* [0.063]
Panel B: Including Groups								
Small Funds	Yes	Yes	No	No	Yes	No	Yes	No
Small Stocks	Yes	No	Yes	No	Yes	Yes	No	No
Growth Stocks	Yes	No	No	Yes	No	Yes	Yes	No
Equal Weight	0.45%*** [0.008]	0.26%* [0.015]	0.38%*** [0.001]	0.33%* [0.078]	0.38%*** [0.001]	0.46%*** [0.009]	0.34%* [0.067]	0.28%*** [0.008]
TNA Weight	0.46%*** [0.004]	0.29%** [0.013]	0.32%** [0.012]	0.39%** [0.024]	0.32%** [0.011]	0.49%*** [0.003]	0.43%** [0.014]	0.26%*** [0.025]

**Table 2.4: The Characteristics of Low and High Volatility Mutual Funds**

This table shows average fund characteristics for the year prior to being sorted into either the low or high volatility portfolio. The low (high) volatility portfolio buys the 10% of mutual funds in the sample with the lowest (highest) standard deviation of daily returns in the prior calendar year. I also present results for the full sample of funds. Panel A shows some average fund level information, and Panel B provides average fund level Fama-French four-factor exposures. Annual Return is the net fund return over the past year. Daily St. Dev. is the standard deviation of daily returns over the past year. Daily Idio. St. Dev. is the standard deviation of the daily Fama-French four-factor residuals over the past year. Assets are the net assets of the fund in millions of dollars. Age is the number of months since the fund started its first share class. Expense is the expense ratio of the fund. Turnover is the turnover ratio of the fund. Beta, SMB, HML, and UMD are the Fama-French four-factor exposures of the fund over the past year estimated from daily returns. Annualized Alpha is the annualized (250 days) Fama-French four-factor alpha over the past year. A  $p$ -value from a test of differences in mean is provided for each characteristic.

Panel A: Fund Level Characteristics					
	Low	High	Difference	$p$ -value	Full Sample
Annual Return	7.41%	10.23%	-2.82%	0.011	7.73%
Daily St. Dev.	0.98%	1.82%	-0.85%	<.001	1.33%
Daily Idio. St. Dev.	0.28%	0.51%	-0.23%	<.001	0.33%
Assets (Millions)	2970	901	2068	<.001	1627
Age (Months)	198	149	49	<.001	179
Expense	1.19%	1.37%	-0.18%	<.001	1.23%
Turnover	60.4%	118.9%	-58.5%	<.001	86.8%
Observations	1485	1474			14792

Panel B: Fund Level Fama-French Exposures					
	Low	High	Difference	$p$ -value	Full Sample
Beta	0.87	1.13	-0.26	<.001	1.01
SMB	0.07	0.63	-0.56	<.001	0.22
HML	0.13	-0.11	0.24	<.001	0.01
UMD	-0.03	0.09	-0.12	<.001	0.03
Annualized Alpha	0.07%	-1.14%	1.21%	.002	-0.61%
Observations	1485	1474			14792

**Table 2.5: Does Fund Volatility Predict Future Performance?**

This table presents results from the following panel model:

$$\text{Alpha}_{i,t+1} = \text{Alpha}_{i,t} + \text{SD}_{i,t} + \text{Idio}_{i,t} + \text{Fund Controls}_{i,t} + \text{Obj FE} + \text{Time FE} + \epsilon_{i,t}$$

The dependent variable is the annualized (250 day) percentage alpha for fund  $i$  in calendar year  $t+1$  calculated from the Fama-French four-factor model using daily returns. SD (Daily St. Dev.) is the standard deviation of daily returns during calendar year  $t$ . Idio (Daily Idio. St. Dev.) is the idiosyncratic standard deviation of daily returns during calendar year  $t$ . Fund Controls include the natural log of fund assets, natural log of age, expense ratio, and turnover ratio all measured as of the end of calendar year  $t$  and the Fama-French four-factor exposures calculated from daily returns during calendar year  $t$ . I include year fixed effects and Lipper objective and class fixed effects. All continuous variables are winsorized at the .5% and 99.5% levels. All continuous right-hand side variables are z-scored, i.e., demeaned and divided by their standard deviation.  $p$ -values from bootstrapped standard errors clustered on year are reported below the coefficients in brackets. \*, \*\*, and \*\*\* represent statistical significance at the 10%, 5%, and 1% levels.

	(1)	(2)	(3)	(4)
Alpha	0.51 [0.239]			0.50 [0.231]
Daily St. Dev.		-2.55* [0.069]		-2.53* [0.092]
Daily Idio. St. Dev.			0.03 [0.963]	
Assets	-0.43*** [0.010]	-0.34** [0.022]	-0.41** [0.021]	-0.36** [0.019]
Age	0.25*** [0.001]	0.18** [0.018]	0.23*** [0.008]	0.21*** [0.002]
Expense	-0.38*** [0.000]	-0.35*** [0.001]	-0.40*** [0.000]	-0.34*** [0.002]
Turnover	-0.38** [0.021]	-0.41** [0.019]	-0.43** [0.017]	-0.36** [0.025]
Beta	-0.65** [0.043]	-0.14 [0.741]	-0.69** [0.026]	-0.11 [0.787]
SMB	-0.17 [0.867]	0.44 [0.619]	-0.07 [0.949]	0.32 [0.695]
HML	1.40** [0.025]	0.83 [0.227]	1.38** [0.021]	0.86 [0.220]
UMD	-0.57 [0.241]	-0.54 [0.292]	-0.52 [0.322]	-0.59 [0.235]
Observations	14,792	14,792	14,792	14,792
Adjusted $r^2$	0.079	0.090	0.074	0.095

**Table 2.6: Does Stock Selectivity Explain the Return on Low Volatility Funds?**

This table shows the monthly percentage alpha for portfolios sorted on one of two measures of stock selectivity (Active Share and  $r^2$ ) and return volatility. Each month funds are first sorted into quintiles based on either (1) Active Share or (2)  $r^2$ . Active Share for a fund is measured following Petajisto (2013). I use the most recent value for Active Share available unless that value is more than three months old. A fund whose most recent value of Active Share is more than three months old is ineligible for inclusion in the portfolios that month. The  $r^2$  for a fund is equal to the  $r^2$  value resulting from the regression of fund monthly returns against the Fama-French four-factor model over the prior twenty four months. At least the prior twelve months of returns are required. After that sort, funds are then sorted within those quintiles into quintiles based on the standard deviation of fund daily returns in the prior calendar year. This double sort produces twenty five groups of funds that are used to form twenty five equal weighted portfolios. Alpha for the portfolios is measured from January 2001 through December 2009 using the Fama-French four-factor model (FF4). The All column and row are portfolios formed on only one of the two groupings after the original sorting procedure has occurred. \*, \*\*, and \*\*\* represent statistical significance using robust standard errors at the 10%, 5%, and 1% levels.

Panel A: Active Share Double Sort

St. Dev. Rank	Active Share Rank					L-H	All
	Low	2	3	4	High		
Low	-0.08	-0.09	-0.03	0.06	0.10	-0.18**	-0.01
2	-0.12***	-0.10*	-0.02	-0.02	0.01	-0.13	-0.05
3	-0.21***	-0.13***	-0.07	-0.13	-0.11	-0.11	-0.13***
4	-0.22***	-0.21***	-0.27***	-0.28***	-0.03	-0.18*	-0.20***
High	-0.28***	-0.39***	-0.40***	-0.35**	-0.44***	0.16	-0.37***
L-H	0.21	0.30*	0.37**	0.40**	0.54***	-0.33**	0.36**
All	-0.18***	-0.18***	-0.16***	-0.14*	-0.09	-0.09	-0.15***

Panel B:  $r^2$  Double Sort

St. Dev. Rank	$r^2$ Rank					L-H	All
	Low	2	3	4	High		
Low	0.09	0.03	0.01	0.03	-0.06	0.15**	0.02
2	-0.06	-0.03	-0.08	-0.15***	-0.11***	0.05	-0.08
3	-0.06	-0.14**	-0.12*	-0.10	-0.24***	0.18	-0.14***
4	-0.02	-0.13*	-0.32***	-0.28***	-0.29***	0.27	-0.21***
High	-0.33**	-0.35***	-0.38***	-0.35***	-0.39***	0.06	-0.36***
L-H	0.41**	0.38**	0.39**	0.39**	0.33**	0.08	0.38***
All	-0.08	-0.12**	-0.18***	-0.17***	-0.22***	0.14	-0.15***

**Table 2.7: How Do Simulated Funds That Invest in High or Low Volatility Stocks Perform?**

This table shows the performance of simulated mutual funds formed on the basis of stock return volatility. To form the simulated funds I first sort all stocks that pass my screens into deciles at the beginning of every year based on the standard deviation of their monthly returns over the previous calendar year. I then randomly choose 50 stocks in the lowest decile of standard deviation and hold them in an equal, value, or randomly weighted portfolio. The same stocks remain in the portfolio for the full calendar year unless they fail a screen or leave the sample. At the beginning of the next year, 50 new stocks are chosen. Value weighted portfolios use the market capitalization of stocks to generate weights and randomly weighted portfolios use the same market capitalization weights each month but randomly assign them to stocks. I follow this procedure to form 1000 low volatility groups that provide 1000 portfolios of each weighting. I then repeat this process using only stocks in highest decile of standard deviation. I measure each simulated fund's performance from January 2000 through December 2011, and average the results for each group. Average Return is the arithmetic average monthly return for the portfolio multiplied by twelve. Geometric Return is the monthly compound return for the portfolio compounded over twelve months. Standard Deviation is the annualized standard deviation of monthly portfolio returns. Sharpe (Treynor) Ratio is the average of the monthly returns less the risk-free rate divided by the portfolio standard deviation (CAPM beta). A *p*-value from a test of differences in mean is provided for each characteristic. Panels A and B present the equal and value weighted results. Panel C presents the random weighted results.

Panel A: Equal Weighted Portfolios

	Low Volatility	High Volatility	Difference	<i>p</i> -value
Average Return	10.8%	-1.5%	12.3%	<.001
Geometric Return	10.5%	-11.1%	21.5%	<.001
SD of Returns	12.5%	44.8%	-32.4%	<.001
Sharpe Ratio	0.68	-0.09	0.77	<.001
Treynor Ratio	0.16	-0.02	0.18	<.001

Panel B: Value Weighted Portfolios

	Low Volatility	High Volatility	Difference	<i>p</i> -value
Average Return	7.0%	-7.4%	14.4%	<.001
Geometric Return	6.2%	-17.2%	23.4%	<.001
SD of Returns	13.8%	47.1%	-33.2%	<.001
Sharpe Ratio	0.34	-0.21	0.55	<.001
Treynor Ratio	0.10	-0.04	0.14	<.001

Panel C: Random Weighted Portfolios

	Low Volatility	High Volatility	Difference	<i>p</i> -value
Average Return	10.8%	-1.7%	12.5%	<.001
Geometric Return	10.1%	-12.6%	22.7%	<.001
SD of Returns	15.5%	48.6%	-33.2%	<.001
Sharpe Ratio	0.55	-0.09	0.64	<.001
Treynor Ratio	0.16	-0.02	0.18	<.001



**Table 2.8: Characteristics of the Low Volatility Minus High Volatility (LVmHV) Factor**

This table presents summary statistics and correlations for the Fama-French four-factors, the Pastor and Stambaugh (2003) liquidity factor, and my LVmHV (low volatility minus high volatility) factor from January 2000 through December 2011. The LVmHV factor is equal to the return to a value weighted portfolio of all stocks that pass my screens that are in the lowest decile of standard deviation of monthly returns during the previous calendar year less the return to a value weighted portfolio of all stocks that pass my screens in the highest decile. Panel A reports the mean and median monthly return for each factor, the standard deviation of the monthly factor returns, and the 10<sup>th</sup> and 90<sup>th</sup> percentile factor returns. Panel B reports the correlations between the monthly returns for each factor.

Panel A: Factor Summary Statistics

Factor	Mean	Median	St. Dev.	10%	90%
LVmHV	1.16%	0.16%	11.66%	-9.97%	13.96%
Mktrf	0.06%	0.76%	4.98%	-7.16%	6.26%
SMB	0.46%	0.08%	3.79%	-3.24%	4.34%
HML	0.53%	0.34%	3.63%	-2.93%	4.39%
UMD	0.17%	0.41%	6.32%	-6.85%	6.19%
PS Liquidity	0.91%	0.71%	4.31%	-4.17%	5.44%

Panel B: Factor Correlations

	LVmHV	Mktrf	SMB	HML	UMD	PS Liq
LVmHV	1					
Mktrf	-0.71	1				
SMB	-0.66	0.30	1			
HML	0.52	-0.11	-0.37	1		
UMD	0.24	-0.38	0.13	-0.10	1	
PS Liquidity	-0.05	0.11	0.13	-0.14	0.08	1

**Table 2.9: Does the Low Volatility Anomaly Explain the Difference in Performance Between Low and High Volatility Funds?**

This table replicates the Fama-French factor results of Table 2.2, but adds new variables to the model. Models (1) through (3) analyze the low volatility portfolio, (4) through (6) analyze the high volatility portfolio, and (7) through (9) analyze the differences between the low and high volatility portfolios. The first new factor added is the LVmHV (low volatility minus high volatility) factor. The LVmHV factor is equal to the return to a value weighted portfolio of all stocks that pass my screens that are in the lowest decile of standard deviation of monthly returns during the previous calendar year less the return to a value weighted portfolio of all stocks that pass my screens in the highest decile. The second new factor is the Pastor and Stambaugh (2003) liquidity factor. *p*-values from robust standard errors are reported below the coefficients in brackets. \*, \*\*, and \*\*\* represent statistical significance at the 10%, 5%, and 1% levels.

	Low Volatility			High Volatility			Low - High		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Beta	0.79*** [0.000]	0.92*** [0.000]	0.90*** [0.000]	1.24*** [0.000]	0.99*** [0.000]	1.01*** [0.000]	-0.45*** [0.000]	-0.08* [0.057]	-0.11*** [0.006]
SMB	0.08*** [0.000]	0.22*** [0.000]	0.20*** [0.000]	0.54*** [0.000]	0.28*** [0.000]	0.29*** [0.000]	-0.46*** [0.000]	-0.06 [0.385]	-0.09 [0.197]
HML	0.25*** [0.000]	0.14*** [0.000]	0.15*** [0.000]	-0.28*** [0.000]	-0.06 [0.226]	-0.07 [0.136]	0.54*** [0.000]	0.20*** [0.001]	0.23*** [0.000]
UMD	0.02 [0.356]	-0.01 [0.505]	-0.01 [0.453]	0.04 [0.479]	0.09*** [0.010]	0.09*** [0.008]	-0.02 [0.758]	-0.10*** [0.003]	-0.10*** [0.001]
LVmHV		0.11*** [0.000]	0.10*** [0.000]		-0.21*** [0.000]	-0.20*** [0.000]		0.32*** [0.000]	0.30*** [0.000]
PS Liquidity			0.04** [0.017]			-0.04 [0.101]			0.09*** [0.002]
Alpha	0.16%** [0.044]	0.03% [0.669]	0.00% [0.982]	-0.30%** [0.020]	-0.05% [0.625]	-0.02% [0.879]	0.45%*** [0.008]	0.07% [0.519]	0.01% [0.902]
Observations	144	144	144	144	144	144	144	144	144
Adjusted r <sup>2</sup>	0.94	0.95	0.96	0.95	0.97	0.97	0.79	0.90	0.91

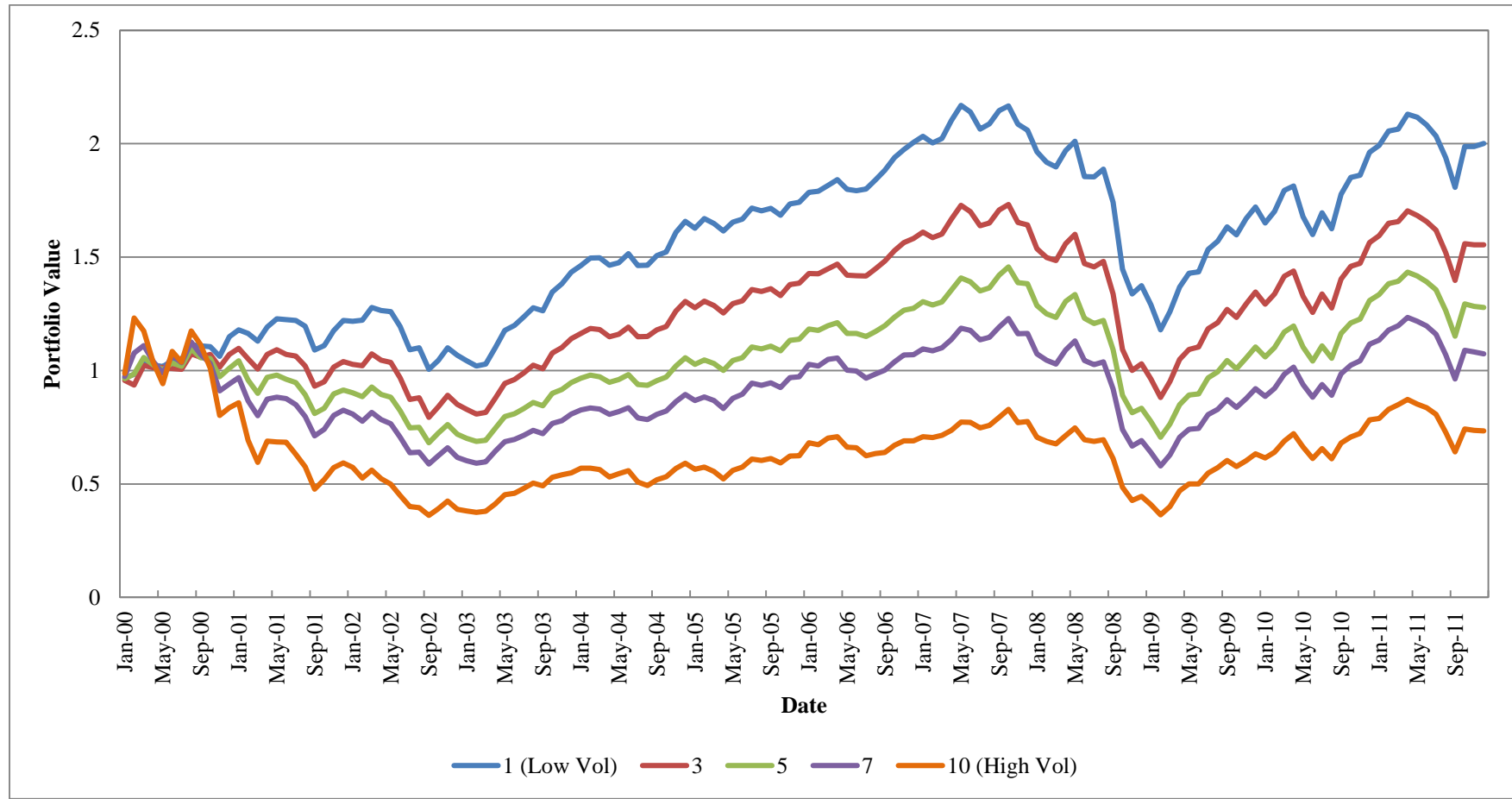
**Table 2.10: How Does the LVmHV Factor Affect Idiosyncratic Volatility?**

This table shows measurements of idiosyncratic volatility for portfolios of low and high volatility mutual funds from January 2000 through December 2011. The low (high) volatility portfolio buys the 10% of mutual funds in the sample with the lowest (highest) standard deviation of daily returns in the prior calendar year. I report the annualized standard deviation of monthly returns for each portfolio and two measures of idiosyncratic volatility. FF4 measures the idiosyncratic volatility as the annualized standard deviation of the residuals from a Fama-French four-factor regression using monthly returns. LVmHV mimics that approach but includes the LVmHV (low volatility minus high volatility) factor in the Fama-French regression. The LVmHV factor is equal to the return to a value weighted portfolio of all stocks that pass my screens that are in the lowest decile of standard deviation of monthly returns during the previous calendar year less the return to a value weighted portfolio of all stocks that pass my screens in the highest decile. Change in Idio. Volatility is the percentage change in idiosyncratic volatility that results from including the LVmHV factor.

Portfolio	Standard Deviation	Idiosyncratic Volatility		Change in Idio. Volatility
		FF4	FF4 + LVmHV	
Low	14.18%	3.33%	2.81%	-15.75%
2	15.27%	3.20%	2.71%	-15.36%
3	15.96%	3.32%	2.94%	-11.40%
4	16.31%	2.70%	2.45%	-9.22%
5	16.86%	2.29%	2.21%	-3.42%
6	17.75%	2.24%	2.24%	-0.10%
7	18.84%	2.25%	2.22%	-1.12%
8	20.28%	2.76%	2.62%	-5.10%
9	21.83%	3.55%	3.13%	-11.85%
High	25.84%	5.52%	4.24%	-23.12%

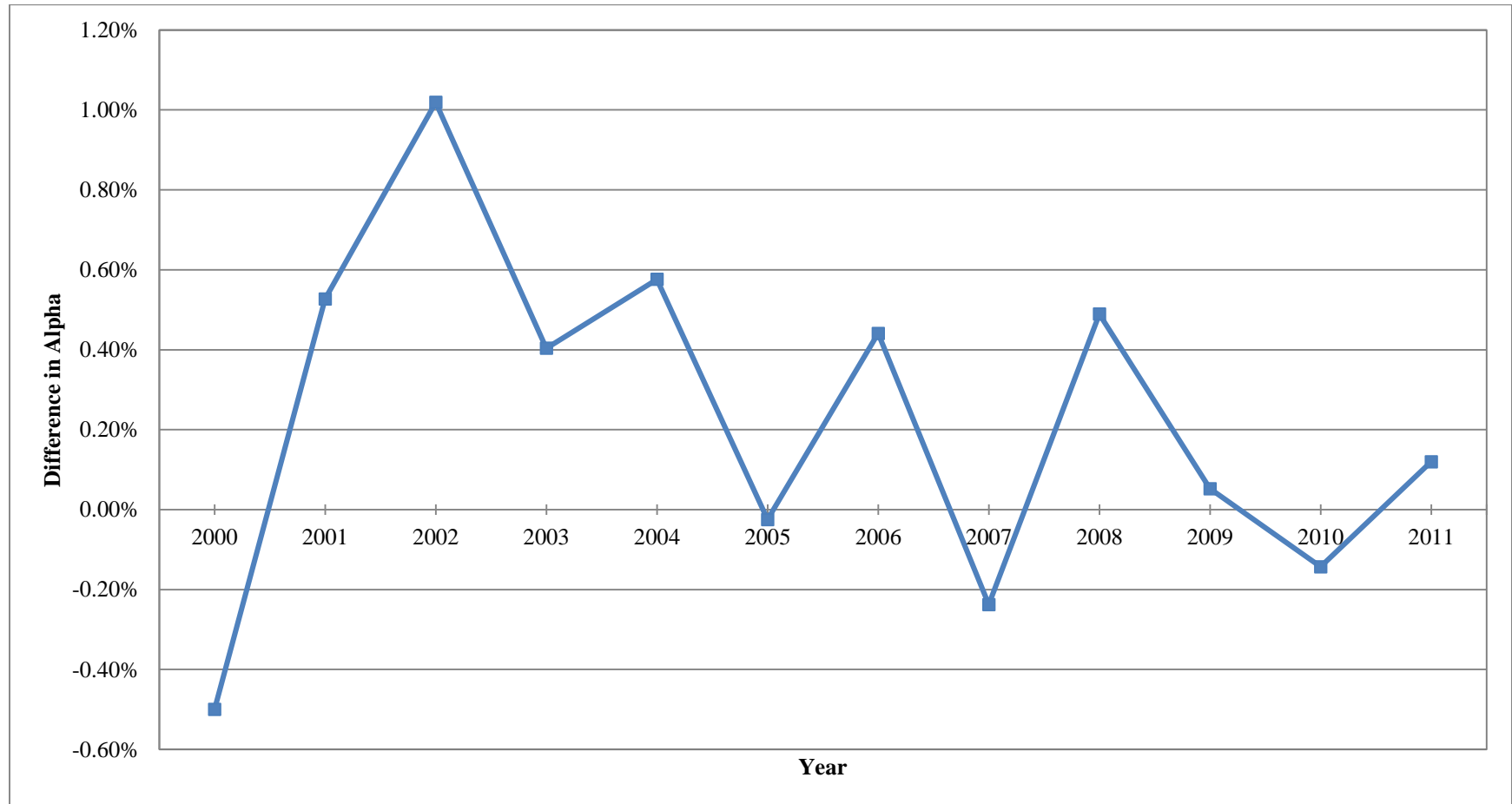
### Figure 2.1: The Return on One Dollar Invested in Mutual Funds Sorted on Past Return Volatility

This figure shows the changing value of \$1.00 invested in January 2000 through December 2011 in five equal weighted portfolios of active U.S. equity mutual funds. The low (high) volatility portfolio buys the 10% of mutual funds in the sample with the lowest (highest) standard deviation of daily returns in the prior calendar year. I only present the 1<sup>st</sup> (low volatility), 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup>, and 10<sup>th</sup> (high volatility) deciles.

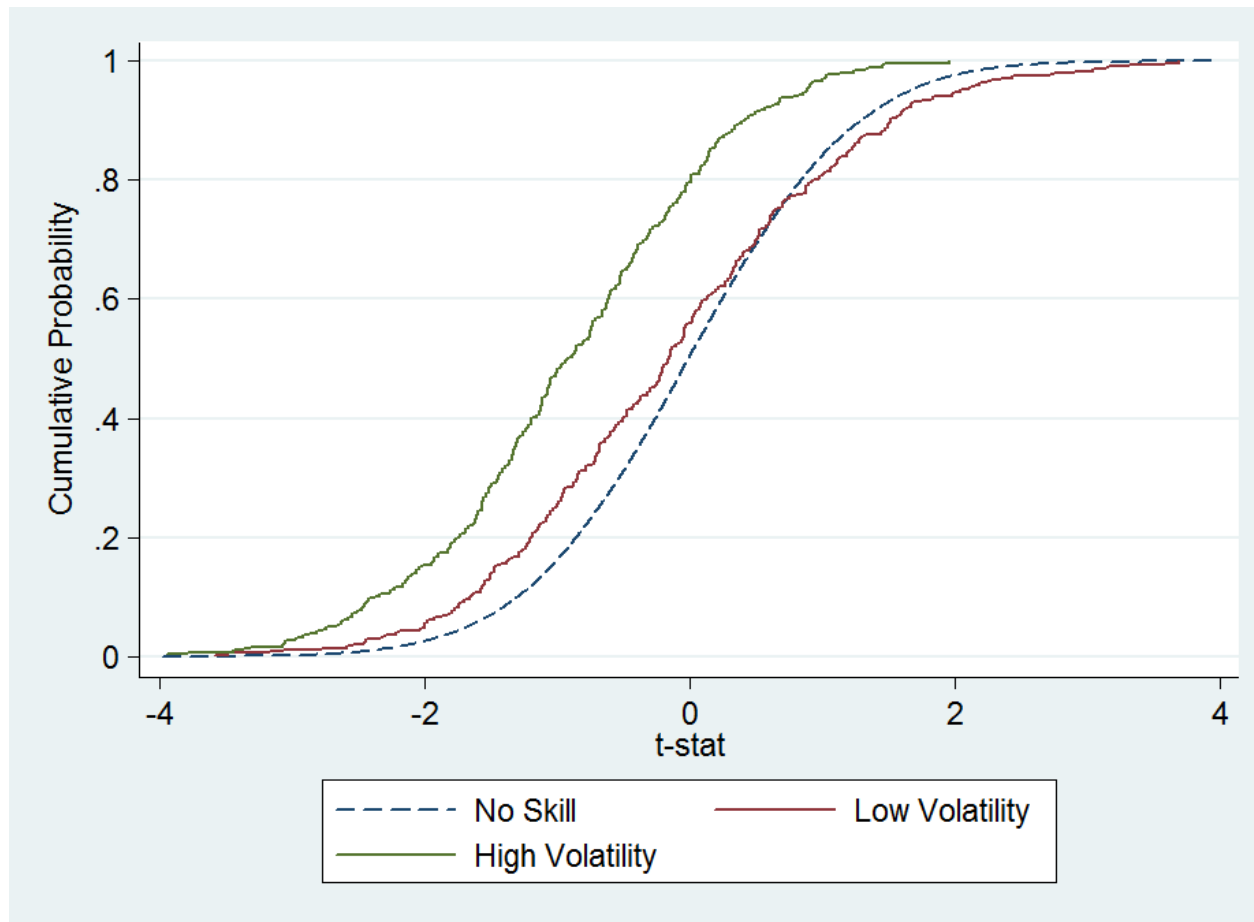


**Figure 2.2: The Difference in Performance Between Low and High Volatility Mutual Funds by Year**

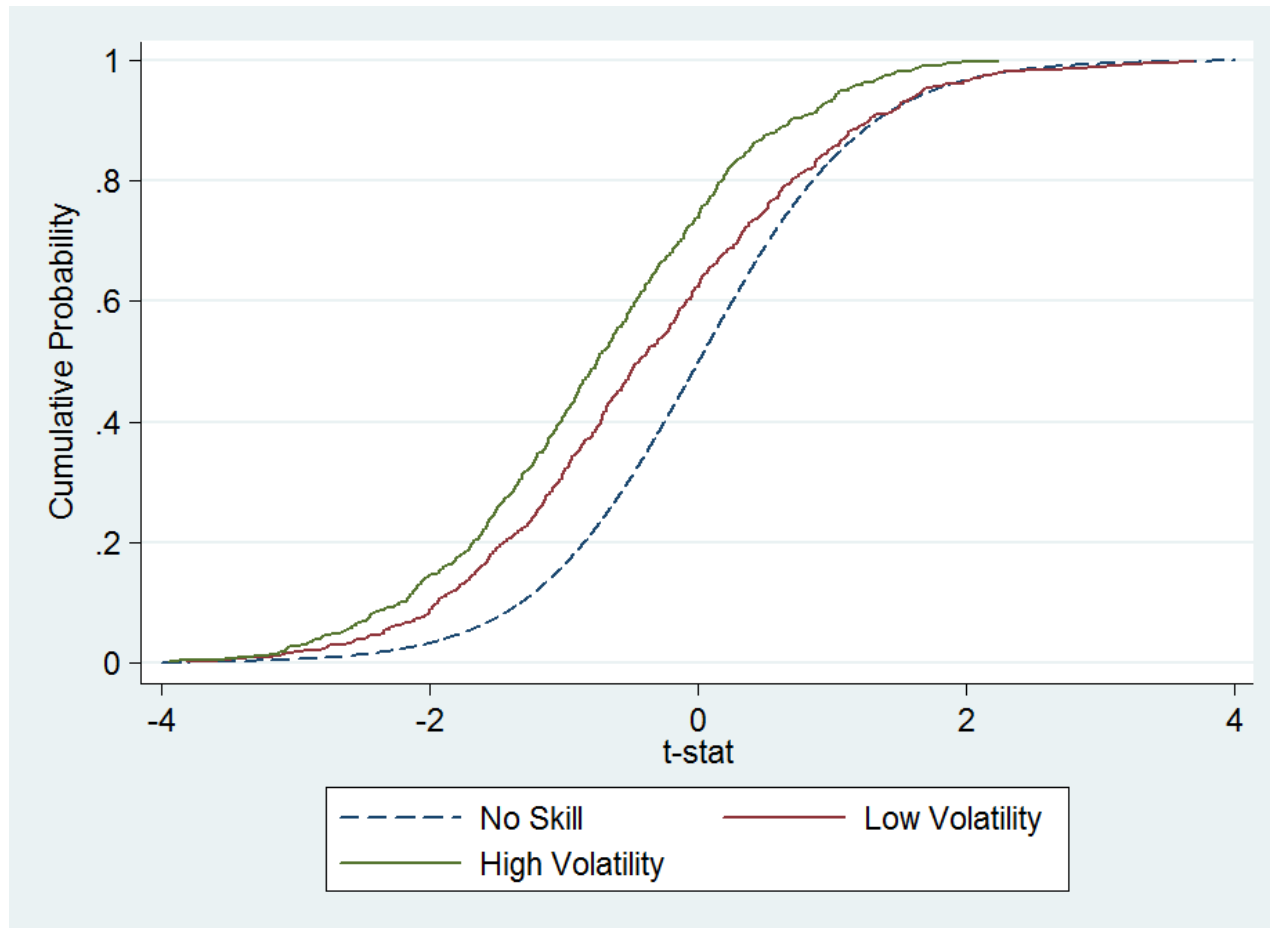
This figure shows the difference in monthly percentage alpha between the low and high volatility portfolios from Table 2.2 each year from 2000 through 2011. The Fama-French four-factor model is used to calculate alpha.



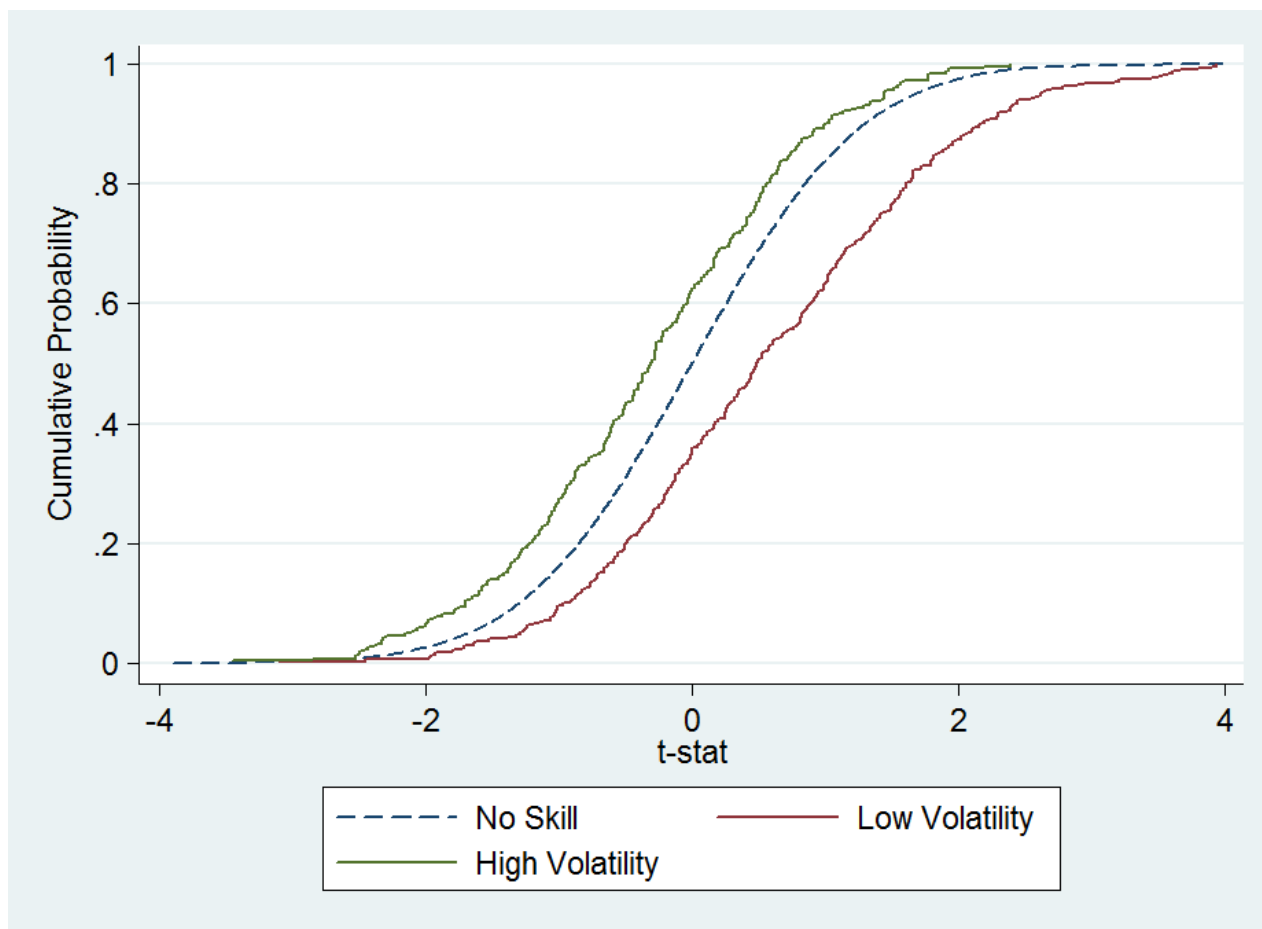
**Figure 2.3A: How Does the Distribution of Mutual Fund Alpha Differ with Respect to Fund Volatility?**  
Net Returns - KTWW Method - FF Alpha



**Figure 2.3B: How Does the Distribution of Mutual Fund Alpha Differ with Respect to Fund Volatility?**  
Net Returns - FF Method - FF Alpha

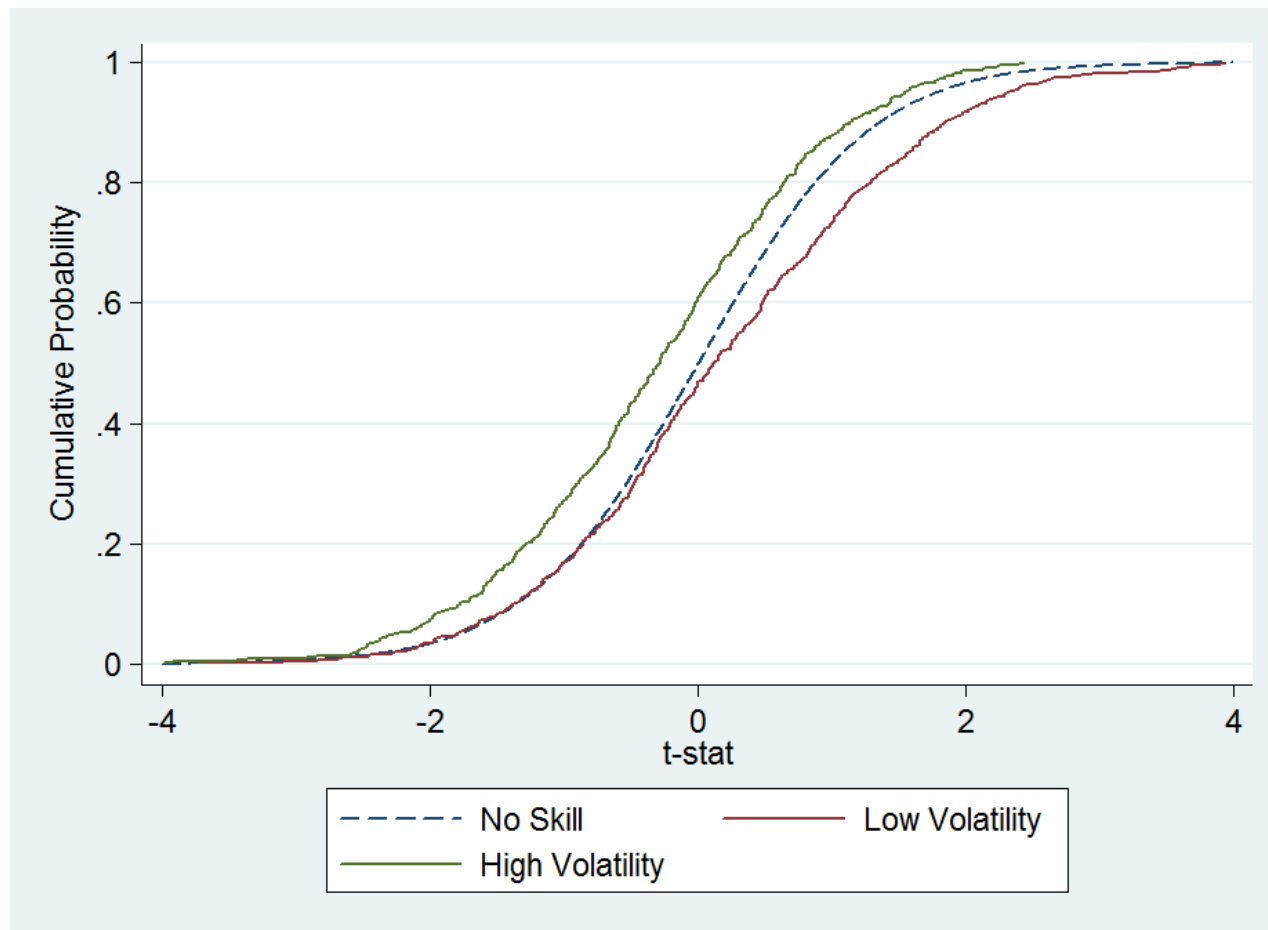


**Figure 2.3C: How Does the Distribution of Mutual Fund Alpha Differ with Respect to Fund Volatility?**  
Gross Returns - KTW Method - FF Alpha





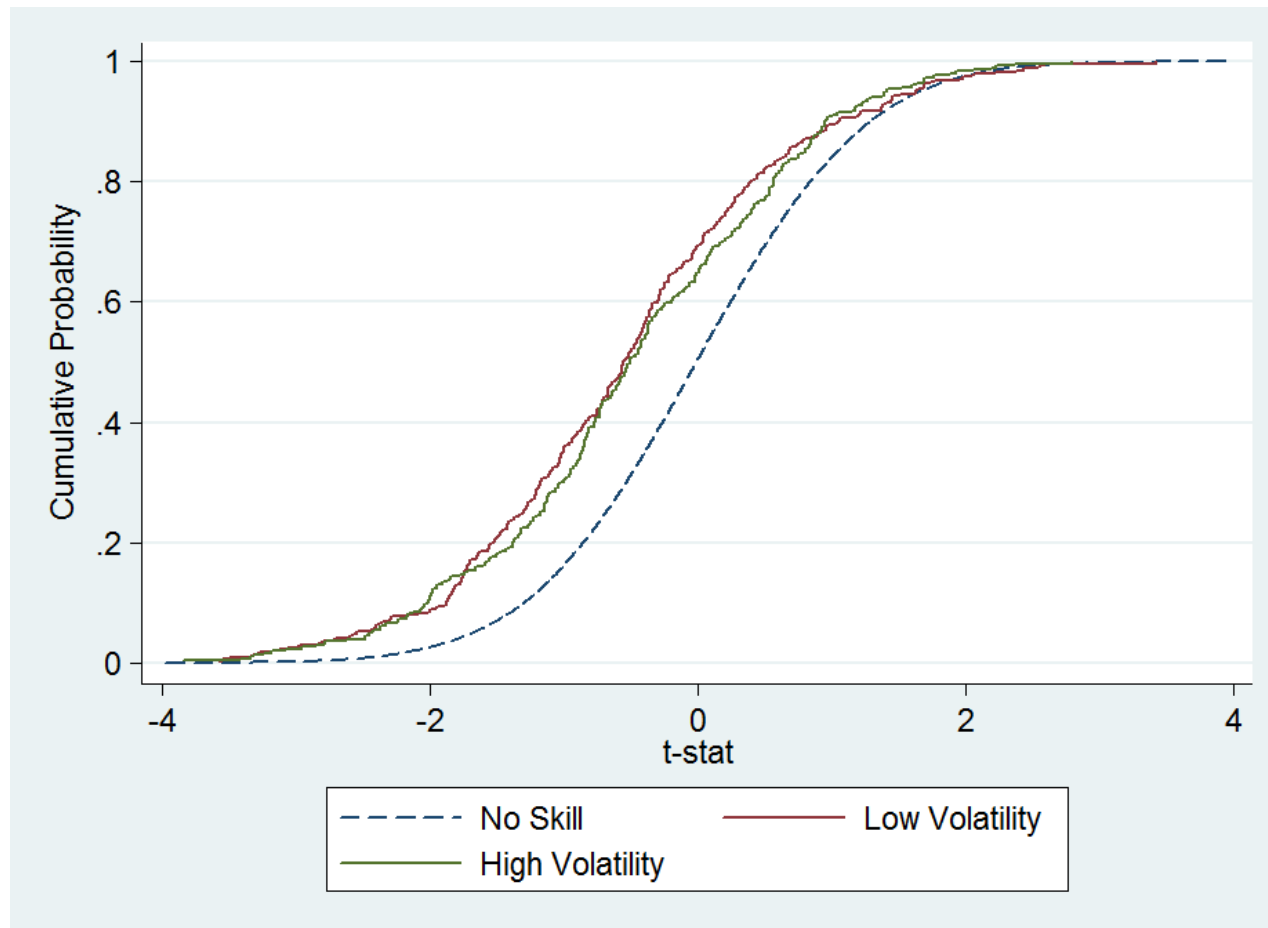
**Figure 2.3D: How Does the Distribution of Mutual Fund Alpha Differ with Respect to Fund Volatility?**  
Gross Returns - FF Method - FF Alpha



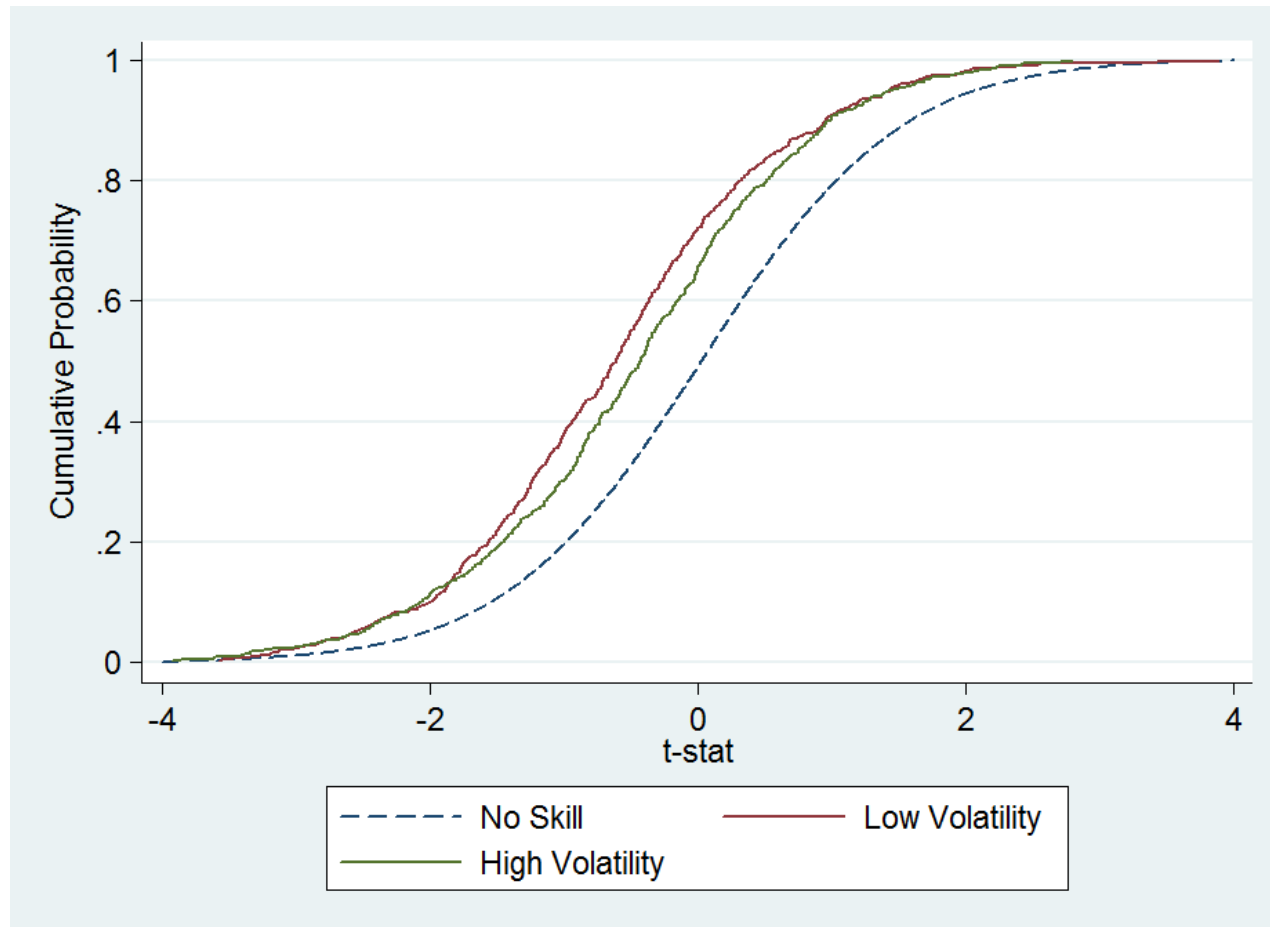
**Figure 3: How Does the Distribution of Mutual Fund Alpha Differ with Respect to Fund Volatility?**

These figures show (1) a plot of the cumulative distribution of the alpha of low volatility mutual funds, (2) a plot of the cumulative distribution of the alpha of high volatility mutual funds, and (3) a combined cumulative distribution calculated under the restriction that fund managers have no skill. A mutual fund is considered low (high) volatility in every month after the standard deviation of its daily returns first falls into the lowest (highest) 10% among funds in the prior calendar year. Only fund months between January 2000 and December 2011 and after a fund is labeled low or high volatility are used. I follow either the Kosowski, Timmerman, Wermers, and White (KTWW) (2006) or the Fama-French (FF) (2010) bootstrap procedure with one thousand simulations to calculate alpha for the low and high volatility funds under the restriction of no manager skill. The KTWW (FF) method requires a fund to have at least sixty (eight) months of returns. Figure 2.3A presents the distributions using net fund returns, the Fama-French four-factor model, and the KTWW bootstrap method. Figure 2.3B presents the distributions using net fund returns, the Fama-French four-factor model, and the Fama-French bootstrap method. Figure 2.3C presents the distributions using gross fund returns, the Fama-French four-factor model, and the KTWW bootstrap method. Figure 2.3D presents the distributions using gross returns, the Fama-French four-factor model, and the Fama-French bootstrap method. I define a fund's gross return for a month as the net return plus one twelfth the expense ratio. Following KTWW and Fama-French, I use the  $t$ -statistic associated with the measurement of alpha rather than alpha itself.

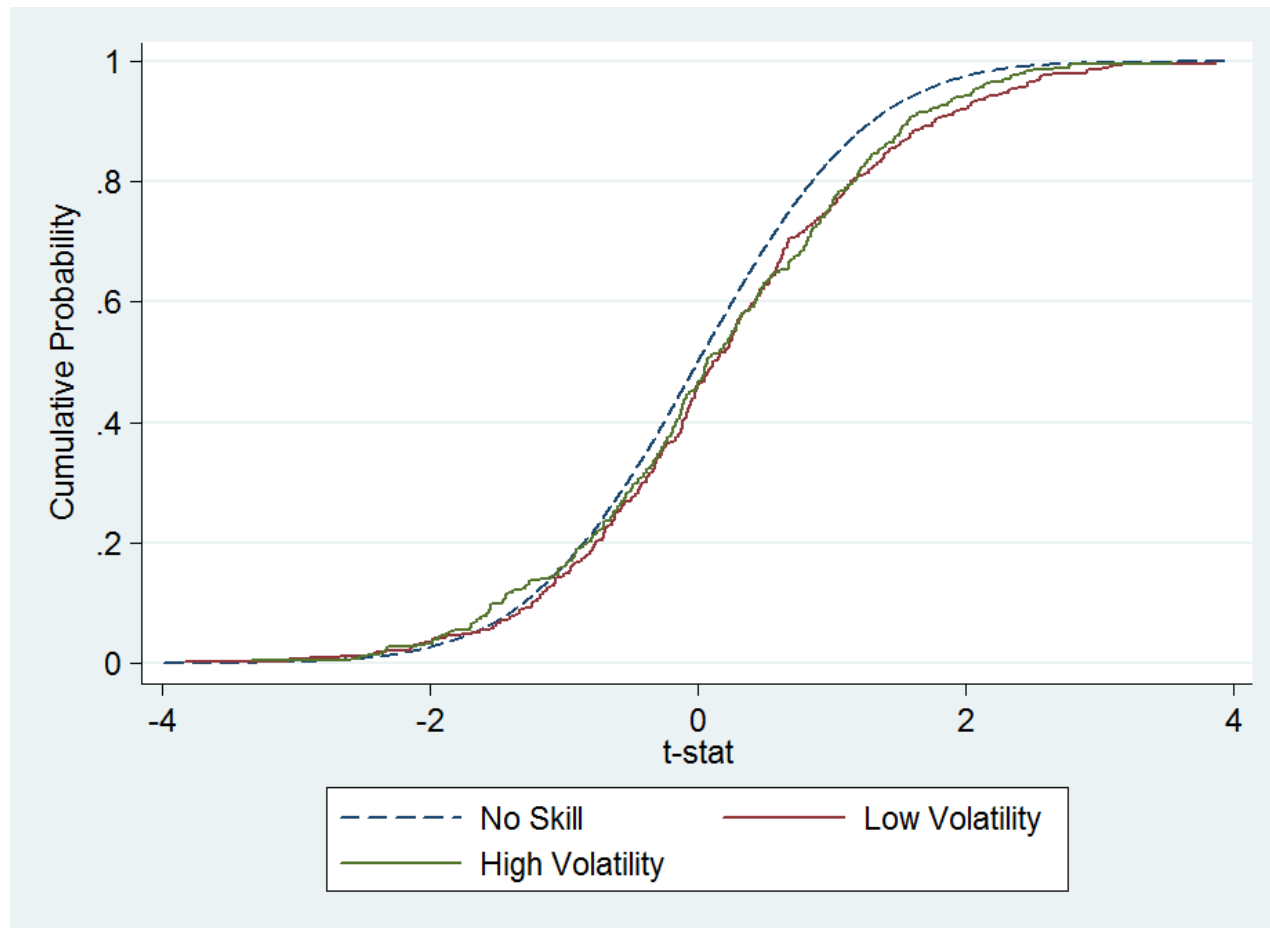
**Figure 2.4A: How Is the Distribution of Mutual Fund Alpha Affected by Accounting for the Low Volatility Anomaly?  
Net Returns - KTWW Method - LVmHV Alpha**



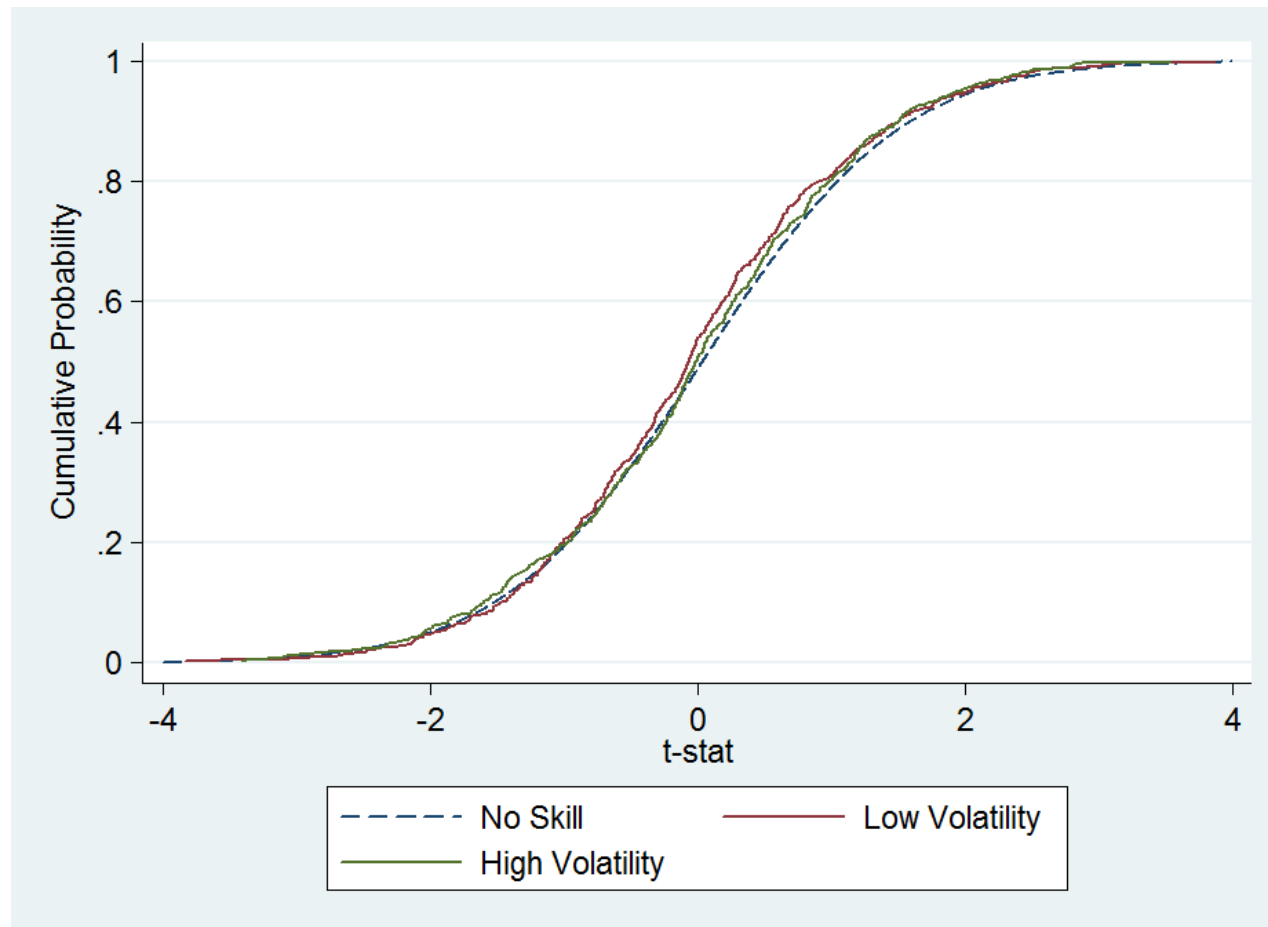
**Figure 2.4B: How Is the Distribution of Mutual Fund Alpha Affected by Accounting for the Low Volatility Anomaly?  
Net Returns - FF Method - LVmHV Alpha**



**Figure 2.4C: How Is the Distribution of Mutual Fund Alpha Affected by Accounting for the Low Volatility Anomaly?  
Gross Returns - KTWW Method - LVmHV Alpha**



**Figure 2.4D: How Is the Distribution of Mutual Fund Alpha Affected by Accounting for the Low Volatility Anomaly?  
Gross Returns – FF Method - LVmHV Alpha**



**Figure 4: How Is the Distribution of Mutual Fund Alpha Affected by Accounting for the Low Volatility Anomaly?**

These figures show (1) a plot of the cumulative distribution of the alpha of low volatility mutual funds, (2) a plot of the cumulative distribution of the alpha of high volatility mutual funds, and (3) a combined cumulative distribution calculated under the restriction that fund managers have no skill. The analysis is identical to Figure 3 except that now I include my LVmHV (low volatility minus high volatility) factor in the Fama-French four-factor model. The LVmHV factor is equal to the return to a value weighted portfolio of all stocks that pass my screens that are in the lowest decile of standard deviation of monthly returns during the previous calendar year less the return to a value weighted portfolio of all stocks that pass my screens in the highest decile. Figure 2.4A presents the distributions using net fund returns, the Fama-French four-factor model augmented with my LVmHV factor, and the KTWW bootstrap method. Figure 2.4B presents the distributions using net fund returns, the Fama-French four-factor model augmented with my LVmHV factor, and the Fama-French bootstrap method. Figure 2.4C presents the distributions using gross fund returns, the Fama-French four-factor model augmented with my LVmHV factor, and the KTWW bootstrap method. Figure 2.4AD presents the distributions using gross returns, the Fama-French four-factor model augmented with my LVmHV factor, and the Fama-French bootstrap method. I define a fund's gross return for a month as the net return plus one twelfth the expense ratio. Following KTWW and Fama-French, I use the  $t$ -statistic associated with the measurement of alpha rather than alpha itself.

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